

The Supersymmetric Standard Model

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The Standard Model may be included within a supersymmetric theory, postulating new *sparticles* that differ by half-a-unit of spin from their standard model partners, and by a new quantum number called *R-parity*. The lightest one, usually a neutralino, is expected to be stable and a possible candidate for dark matter.

The electroweak breaking requires two doublets, leading to several charged and neutral Brout-Englert-Higgs bosons. This also leads to gauge/Higgs unification by providing extra spin-0 partners for the spin-1 W^\pm and Z . It offers the possibility to view, up to a mixing angle, the new 125 GeV boson as the spin-0 partner of the Z under two supersymmetry transformations, i.e. as a Z that would be deprived of its spin. Supersymmetry then relates *two existing particles of different spins*, in spite of their different gauge symmetry properties, through supersymmetry transformations acting on physical fields in a non-polynomial way.

We also discuss how the compactification of extra dimensions, relying on *R-parity* and other discrete symmetries, may determine both the supersymmetry-breaking and grand-unification scales.

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Is there a superworld of new particles? Could half of the particles at least have escaped our observations? Do new states of matter exist? After the prediction of anti-matter by Dirac, supersymmetric extensions of the standard model lead to anticipate the possible existence of spin-0 squarks and sleptons, with the gluons, W^\pm , Z and photon associated with gluinos, charginos and neutralinos [1–4]. These new states differ from ordinary particles by half-a-unit of spin and are distinguished by a *R-parity* quantum number related to baryon and lepton numbers, making the lightest superpartner stable, and a possible candidate for the dark matter of the Universe. Spontaneous electroweak breaking is induced by two spin-0 doublets instead of one in the standard model, leading to several charged and neutral spin-0 BEH bosons. These may even be related to the massive gauge bosons, with the possibility that the new 125 GeV boson recently discovered at CERN [5, 6] be a spin-0 partner of the Z under *two* supersymmetry transformations [1, 7, 8]. But, where is all this coming from?

I. FUNDAMENTAL INTERACTIONS, SYMMETRY BREAKING AND THE NEW SPIN-0 BOSON

Special relativity and quantum mechanics, operating within quantum field theory, led to the Standard Model of particles and interactions (SM). It has met a long series of successes with the discoveries of weak neutral currents (1973), charmed particles (1974–76), gluons mediators of strong interactions (1979), W^\pm and Z 's mediators of weak interactions (1983), and the sixth quark known as the top quark (1995). Weak, electromagnetic and strong inter-

TABLE I: Particle content of the standard model.

spin-1 gauge bosons :	gluons, W^+ , W^- , Z , photon
spin- $\frac{1}{2}$ fermions :	$\left\{ \begin{array}{l} 6 \text{ quarks: } \begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \\ 6 \text{ leptons: } \begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} \end{array} \right.$
spin-0	scalar BEH boson

actions are well understood from the exchanges of spin-1 mediators between spin- $\frac{1}{2}$ quarks and leptons, generically referred to as the constituents of matter (cf. Table I).

The eight gluons mediate the strong interactions, invariant under the color $SU(3)$ gauge group. The W^\pm , Z and photon are associated with the $SU(2) \times U(1)$ electroweak gauge group [9–12]. The W^\pm and Z masses, $m_W \simeq 80 \text{ GeV}/c^2$ and $m_Z \simeq 91 \text{ GeV}/c^2$, are generated through the spontaneous breaking of the electro-weak symmetry, induced in the standard model by a doublet of spin-0 fields φ [11, 12]. Three of its four real components, instead of being associated with unwanted massless Goldstone bosons [13], are eliminated by the Brout-Englert-Higgs mechanism [14–16] to provide the extra degrees of freedom for the massive W^\pm and Z . The fourth component, taken as $\phi = \sqrt{2} \varphi^\dagger \varphi$, adjusts so that the potential

$$V(\varphi) = \lambda_{\text{SM}} (\varphi^\dagger \varphi)^2 - \mu_{\text{SM}}^2 \varphi^\dagger \varphi \quad (1)$$

is minimum, for $\phi = v = \sqrt{\mu_{\text{SM}}^2 / \lambda_{\text{SM}}}$ [11–15].

The electroweak symmetry, said to be “spontaneously broken”, is in fact simply hidden, with ϕ being gauge-invariant. The W^\pm and Z acquire masses $m_W = gv/2$, $m_Z = \sqrt{g^2 + g'^2} v/2 = m_W/\cos\theta$, with $\tan\theta = g'/g$. The elementary charge and the Fermi coupling of weak interactions are given by $e = g \sin\theta$ and $G_F/\sqrt{2} = g^2/8m_W^2 = 1/2v^2$, so that $v = (G_F\sqrt{2})^{-1/2} \simeq 246$ GeV. Charged lepton and quark fields interact with φ with coupling constants $\lambda_{l,q}$, so that the corresponding particles, sensitive to the physical BEH field $\phi = \sqrt{2}\varphi^\dagger\varphi$ with $\langle\phi\rangle = v$, acquire masses $m_{l,q} = \lambda_{l,q}v/\sqrt{2}$, neutrinos remaining massless at this stage. The waves corresponding to the space-time variations of $\phi = \sqrt{2}\varphi^\dagger\varphi$, when quantized, are associated with spin-0 Brout-Englert-Higgs bosons, commonly referred to as Higgs bosons. Their mass,

$$m_h = \sqrt{2\mu_{\text{SM}}^2} = \sqrt{2\lambda_{\text{SM}}v^2}, \quad (2)$$

is fixed by the quartic coupling λ_{SM} in the scalar potential $V(\varphi)$ in (1), a mass of 125 GeV/ c^2 corresponding to a coupling

$$\lambda_{\text{SM}} = \frac{m_h^2}{2v^2} = \frac{g^2 + g'^2}{8} \frac{m_h^2}{m_Z^2} = \frac{G_F m_h^2}{\sqrt{2}} \simeq 0.13. \quad (3)$$

The possible origin of this coupling will be discussed later, within supersymmetric theories. They lead to consider several BEH bosons originating from the two spin-0 doublets

$$h_1 = \begin{pmatrix} h_1^0 \\ h_1^- \end{pmatrix}, \quad h_2 = \begin{pmatrix} h_2^+ \\ h_2^0 \end{pmatrix}, \quad (4)$$

relating their quartic couplings to the squares of the electroweak gauge couplings, in particular through

$$\text{Supersymmetry} \Rightarrow \lambda_{\text{SM}} \rightarrow \frac{g^2 + g'^2}{8}, \quad (5)$$

with $(g^2 + g'^2)/8 = G_F m_Z^2/\sqrt{2} \simeq .069$. Here we first focus for simplicity on h_2 and on the “large $\tan\beta$ limit”, for which h_2 acquires a non-vanishing v.e.v. much larger than for h_1 . We then get a neutral BEH boson that would have the same mass as the Z [1], according to

$\text{Supersymmetry} \Rightarrow$ $m_h = \sqrt{2\lambda_{\text{SM}}v^2} = \frac{\sqrt{g^2 + g'^2} v}{2} = m_Z \simeq 91 \text{ GeV}/c^2,$

(6)

up to supersymmetry-breaking effects.

This mass equality results from an unbroken supersymmetry in the sector of neutral particles, with the spin-1 Z and the spin-0 h in the same massive multiplet of supersymmetry. It remains valid even *independently of the value of the mixing angle β* defined from the ratio of the two doublet v.e.v.’s by

$$\tan\beta = \frac{v_2}{v_1}, \quad (7)$$

with $\langle h_i^0 \rangle = v_i/\sqrt{2}$, as long as supersymmetry remains unbroken in this sector [1]. The corresponding spin-0 boson then appears as *the spin-0 partner of the Z under two supersymmetry transformations* [7, 8]. It was even originally denoted by z to make this association explicit.

Finding such a spin-0 boson with a mass of 125 GeV/ c^2 [5, 6], not much higher than the Z mass, may thus be considered, at least, as a very encouraging sign for supersymmetry. This is especially true as the value $m_h \simeq m_Z$ required by unbroken supersymmetry may be increased up to 125 GeV/ c^2 by supersymmetry-breaking effects. This is the case, most notably, in models such as the N/nMSSM or USSM, that include an extra singlet next to the two doublets in (4), with a trilinear $\lambda H_2 H_1 S$ superpotential coupling [1].

The lightest spin-0 mass may then easily reach 125 GeV, without having to rely on very large effects from radiative corrections. This is a much better situation than in the usual MSSM for which no electroweak breaking is obtained in the absence of the supersymmetry-breaking terms, m_h is required to be less than m_Z at the classical level, and it is difficult to obtain such a 125 GeV spin-0 boson from sufficiently large radiative corrections involving very heavy stop quarks.

The scalar boson of the standard model has long remained its last missing particle after the discovery of the top quark in 1995. The new boson found at CERN in 2012 shows the properties expected from a scalar boson associated with the differentiation between electromagnetic and weak interactions, and the generation of masses. It may well be identified with the one of the standard model, which may then be considered as complete.

Still, it would be presumptuous to imagine that our knowledge of particles and interactions is now complete, without new particles or interactions remaining to be discovered. The standard model does not answer many fundamental questions, concerning the origin of symmetries and symmetry breaking, the quark and lepton mass spectrum and mixing angles, etc.. Gravitation, classically described by general relativity, cannot easily be cast into a consistent quantum theory. This is why string theories were developed, which seem to require supersymmetry for consistency.

The nature of dark matter and dark energy which govern the evolution of the Universe and its accelerated expansion remains unknown, as the origin of the predominance of matter over antimatter. Dark matter may be composed, for its main part, non-baryonic, of new particles, such as the neutralinos of supersymmetric theories. There may also be new forces or interactions beyond the four known ones. And maybe, beyond space and time, new hidden dimensions, extremely small or even stranger, like the anticommuting quantum dimensions of supersymmetry.

II. INTRODUCING SUPERSYMMETRY

In contrast with pions, kaons and other spin-0 mesons, composed of quarks and antiquarks, the new 125 GeV boson presents at this stage all the characteristics of an elementary spin-0 particle, the first one of its kind. The possible existence of such a scalar has long been questioned, many physicists having serious doubts about the very existence of fundamental spin-0 fields. More specifically in a theory involving very high mass or energy scales much larger than the electroweak scale, such as a grand-unification scale [17, 18] (now usually believed to be of the order of 10^{16} GeV), or the Planck scale $\simeq 10^{19}$ GeV possibly associated with quantum gravity, such spin-0 fields tend to acquire very large mass terms. They would then disappear from the low-energy theory, no longer being available to provide an appropriate breaking of the electroweak symmetry.

Many efforts were thus devoted to replace fundamental spin-0 fields by composite fields built from spin- $\frac{1}{2}$ ones, without however much success at this point. These spin- $\frac{1}{2}$ subconstituent fields could have been, for example, techniquark fields interacting through a new interaction specially introduced for this purpose [19–22], in view of ultimately avoiding fundamental spin-0 fields and particles associated with the electroweak breaking, like the one discovered recently. Furthermore it would still remain difficult to completely avoid considering fundamental spin-0 fields, e.g. to trigger the breaking of the initial extended technicolor gauge group.

In the meantime however, and even before these increased questionings about fundamental spin-0 bosons, the situation concerning our view of spin-0 fields had already changed considerably with the introduction of supersymmetry, in the early 1970's. This one provides a natural framework for fundamental spin-0 fields. They may now be treated on the same footing as spin- $\frac{1}{2}$ ones, also benefiting from the same mass terms when supersymmetry is unbroken; and of mass terms which may remain moderate as compared to very large scales if supersymmetry is not too badly broken, then remaining available to trigger the electroweak breaking.

The supersymmetry algebra involves a self-conjugate (Majorana) spin- $\frac{1}{2}$ generator Q satisfying the anticommutation and commutation relations [23–28]

$$\left\{ \begin{array}{l} \{ Q, \bar{Q} \} = -2\gamma_\mu P^\mu, \\ [Q, P^\mu] = 0. \end{array} \right. \quad (8)$$

They express that supersymmetry transformations may be combined to generate translations, and commute with them. This algebra was originally introduced as a parity-violating one that might help understanding why weak interactions violate parity [23], or the masslessness of a neutrino by trying to view it as a Goldstone fermion [24], two possible motivations that soon appeared inadequate. It may also be obtained by generalizing to 4 dimensions

the algebra of supergauge transformations acting in the 2-dimensional string worldsheet [25].

But what physical implications may really be extracted from the consideration of this algebraic structure? According to common knowledge, supersymmetry should relate bosons, of integer spin, with fermions, of half-integer spin, as follows:

$$\text{supersymmetry} \quad \begin{matrix} \text{bosons} & \longleftrightarrow & \text{fermions} \end{matrix} \quad (9)$$

But even this is not always valid, as there are supersymmetric theories involving only fundamental fermions, with supersymmetry transformations acting in a non-linear way [24]. Strictly speaking the algebraic structure of supersymmetry does not even require any boson at all, not to mention the superpartners that we shall introduce later. But let us leave aside such unconventional situations. Let us add, also, that supersymmetry transformations are usually expected to relate bosons and fermions with the same gauge symmetry properties.

Then, can this algebra be of any help in understanding the real world of particles and interactions? If supersymmetry is to act at the fundamental level the natural idea would be to use it to relate the known bosons and fermions in Table I. More precisely, can one relate the bosons (gluons, W^\pm , Z and photon) messengers of interactions to the fermions, quarks and leptons, constituents of matter? This would lead to a sort of unification

$$\text{supersymmetry ?} \quad \begin{matrix} \text{Forces} & \longleftrightarrow & \text{Matter.} \end{matrix} \quad (10)$$

The idea looks attractive, even so attractive that supersymmetry is frequently presented as uniting forces with matter. This is however misleading at least at the present stage, and things do not work out that way.

Indeed the algebraic structure of supersymmetry did not seem applicable to particle physics at all, in particular as known fundamental bosons and fermions do not seem to have much in common. There are also a number of more technical reasons, dealing with: 1) the difficulties of spontaneous supersymmetry breaking, originating from the presence of the hamiltonian within the algebra; 2) the fate of the resulting Goldstone fermion, after one has succeeded in breaking supersymmetry spontaneously [1, 29–31], and as it may well continue to interact, even after getting eaten away by the spin- $\frac{3}{2}$ gravitino, according to the “equivalence theorem” of supersymmetry [32]; 3) the presence of self-conjugate Majorana fermions, unknown in Nature; 4) the requirements of baryon and lepton number conservation, which got associated with the definition of R -symmetry and the requirement of R -parity, etc..

Relating bosons and fermions, yes, but how? One has to find out which of them might be related under supersymmetry, first considering possible associations between

mesons and baryons. Or, at the fundamental level, exploring as a necessary exercise tentative associations like

$$\left\{ \begin{array}{ccc} \text{photon} & \xleftrightarrow{?} & \text{neutrino} \\ W^\pm & \xleftrightarrow{?} & e^\pm \\ \text{gluons} & \xleftrightarrow{?} & \text{quarks} \\ \dots & & \end{array} \right. \quad (11)$$

But we have no chance to realize in this way systematic associations of known fundamental bosons and fermions. This is also made obvious as we know 90 fermionic field degrees of freedom (for 3 families of 15 chiral quark and lepton fields) as compared to 28 only for bosonic ones ($16 + 11 + 1$ including the new scalar). Furthermore these fields have different gauge and B and L quantum numbers, preventing them from being directly related.

In supersymmetry we also have to deal with the systematic appearance of self-conjugate Majorana fermions, while Nature seems to know Dirac fermions only (with a possible exception for neutrinos with Majorana mass terms). How can we obtain Dirac fermions, and attribute them conserved quantum numbers like B and L ? And if we start attributing B and L also to bosons (now known as squarks and sleptons), how can we be sure that their exchanges won't spoil the B and L conservation laws, at least to a sufficiently good approximation? It is thus far from trivial to consider applying supersymmetry to the real world. But if this program can be realized and if Nature has "chosen" being supersymmetric, consequences promise being spectacular.

Addressing the difficult questions of spontaneous supersymmetry breaking, and electroweak breaking, will lead us, through the definition of a new symmetry called R symmetry with its discrete remnant known as R -parity, to the Supersymmetric Standard Model. The way to see supersymmetry now is to view it as *an extension of the standard model* that introduces a new *sparticle* for each one in the standard model [1–4], in particular through

$$\left\{ \begin{array}{ccc} \text{quarks, leptons} & \leftrightarrow & \text{spin-0 squarks and sleptons,} \\ \text{gluons} & \leftrightarrow & \text{spin-}\frac{1}{2}\text{ gluinos,} \\ W^\pm, Z, \gamma & \leftrightarrow & \text{spin-}\frac{1}{2}\text{ charginos and neutralinos,} \end{array} \right. \quad (12)$$

with more to say about spin-0 BEH bosons, including charged and several neutral ones.

While this is now often presented as obvious, the necessity of postulating that *every known particle has its own image under supersymmetry* (SM bosons having fermionic superpartners and SM fermions bosonic ones) was long mocked as a sign of the irrelevance of supersymmetry. The introduction of a color octet of spin- $\frac{1}{2}$ Majorana fermions called *gluinos* was also, at the time, forbidden by the principle of triality [33]. This one, however, gets systematically violated within supersymmetric theories.

The necessity of *charged spin-0 BEH bosons* (H^\pm), required by the 2-doublet structure of supersymmetric theories, was also taken as an argument against supersymmetry and supersymmetric extensions of the standard model, on the grounds that even a single doublet, although possibly necessary as in the standard model, was already undesirable. These charged spin-0 bosons, which have not been discovered yet [34, 35], also appear as the spin-0 partners of the W^\pm under *two* supersymmetry transformations, very much as the new 125 GeV boson may now also be interpreted as the spin-0 partner of the Z , up to supersymmetry-breaking effects.

III. SUPERSYMMETRY BREAKING AND R SYMMETRY

A. Is spontaneous supersymmetry breaking possible at all?

If bosons and fermions are directly related by supersymmetry they should have equal masses. Supersymmetry may then be only, at best, a broken symmetry. Considering terms breaking explicitly the supersymmetry (as frequently done now) would certainly make the task much easier, but may be considered only as a temporary substitute for a solution to the problem of supersymmetry breaking. If supersymmetry is to be a genuine symmetry for the theory and its equations of motion, it should be broken *spontaneously*, as for the electroweak symmetry in the standard model. This is also necessary for supersymmetry to be realized as a local fermionic gauge symmetry [36]. It must then include general relativity, leading to supergravity theories [37, 38].

To trigger a spontaneous breaking of an ordinary (global or gauge) symmetry, one simply has to arrange for the symmetric vacuum state to be unstable, e.g. by choosing a negative value for the mass² parameter $-\mu_{\text{SM}}^2$ in the potential (1), which is easily realized.

The situation concerning supersymmetry is, however, completely different. The hamiltonien H , which governs the energy of the possible vacuum states and thus determines which one is going to be stable, may now be expressed from the squares of the four components of the supersymmetry generator, as

$$H = \frac{1}{4} \sum_{\alpha} Q_{\alpha}^2. \quad (13)$$

This implies that a *supersymmetric vacuum state* $|\Omega\rangle$ (verifying $Q_{\alpha}|\Omega\rangle = 0$) *must have a vanishing energy*, with $H|\Omega\rangle = 0$. On the other hand any non-supersymmetric state $|\Omega'\rangle$ would have, within global supersymmetry, a larger, positive, energy density, and thus would be unstable. This was originally thought to prevent any spontaneous breaking of the supersymmetry to possibly occur [39], apparently signing the impossibility of applying supersymmetry to the real world.

B. In search for a minimum of the potential breaking the supersymmetry

In spite of this general argument however, which soon got circumvented, spontaneous supersymmetry breaking turned out to be possible, although in very specific circumstances. It is severely constrained and usually hard to obtain, at least within global supersymmetry, as any supersymmetric candidate for the vacuum state ($|\Omega\rangle$) is necessarily stable. Furthermore in the presence of many spin-0 fields, there are usually many opportunities for them to adjust so as to provide such a stable supersymmetric vacuum, with a vanishing value of the potential $V = 0$.

To obtain a spontaneous breaking of the global supersymmetry, one cannot just attempt to make a supersymmetric vacuum unstable. One must instead arrange for such a symmetric state to be *totally absent*, as it would otherwise be stable owing to expression (13) of the hamiltonian.

In the usual langage of global supersymmetry [26, 27] involving gauge superfields $V_a(x, \theta, \bar{\theta})$ and (left-handed) chiral superfields $\Phi_i(x, \theta)$ with physical spin-0 and spin- $\frac{1}{2}$ components ϕ_i and $\tilde{\phi}_{iL}$ [40, 41], the potential of scalar fields is expressed as

$$V = \frac{1}{2} \sum (D_a^2 + F_i^2 + G_i^2) = \sum_a \frac{D_a^2}{2} + \sum_i \left| \frac{\partial \mathcal{W}}{\partial \phi_i} \right|^2. \quad (14)$$

D_a and (F_i, G_i) stand for the auxiliary components of gauge and chiral superfields. The contribution from the D terms is given by

$$V_D = \sum_a \frac{D_a^2}{2} = \frac{1}{2} \sum_a \left[\xi_a + g_a \sum_{ij} \phi_i^* (T_a)_{ij} \phi_j \right]^2, \quad (15)$$

with the ξ_a parameters relative to abelian $U(1)$ factors in the gauge group [29]. The superpotential $\mathcal{W}(\Phi_i)$ is an analytic function of the chiral superfields.

For a supersymmetric vacuum state $|\Omega\rangle$ to be, not unstable but totally absent, the potential V must be *strictly positive everywhere*. One at least of these auxiliary components must then have a non-vanishing v.e.v., which is indeed the signal for a spontaneously broken supersymmetry (except for trivial situations with a free superfield). Finding a spontaneously broken supersymmetry then amounts to finding situations for which the set of equations

$$\langle D_a \rangle = \langle F_i \rangle = \langle G_i \rangle = 0 \text{ must have no solution.} \quad (16)$$

How this may be realized, as well as the definition and role of the R symmetry, leading to R -parity, to appropriately constrain the superpotential, will be further discussed in the rest of this Section. The reader mostly interested in the construction of supersymmetric extensions of the standard model (MSSM, N/nMSSM, USSM,

etc.) and in the relations between massive spin-1 gauge bosons and spin-0 BEH bosons may choose to proceed directly to Sections IV and V.

C. D and F supersymmetry breaking mechanisms, in connection with R symmetry

To avoid having a vanishing minimum of V when all physical fields ϕ_i vanish, there are two possibilities, which may be combined:

1) The Lagrangian density may include a linear term

$$\mathcal{L}_\xi = \xi D, \quad (17)$$

associated with an abelian $U(1)$ factor in the gauge group [1, 29]. This term is indeed supersymmetric, up to a derivative which does not contribute to the variation of the action integral, and gauge invariant for an abelian gauge group. It may lead to a spontaneous breaking of the supersymmetry, just by itself as in the presence of a single chiral superfield S (with a charge e such that $\xi e > 0$) [42], or by making the set of equations $\{D_a = 0\}$ without solution, as with a $SU(2) \times U(1)$ gauge group [1]. The Goldstone spinor is then a gaugino, corresponding for example to the photino in a $SU(2) \times U(1)$ theory, even if such a feature cannot persist in a physically realistic theory [4].

One may also arrange so that the set of equations $\{D_a = 0, F_i = G_i = 0\}$ be without solution, as done in the presence of chiral superfields S and \bar{S} with a mass term $\mu S \bar{S}$ [29]; or with a suitable trilinear superpotential $\lambda H_2 H_1 S$, the electroweak gauge group being extended to an extra $U(1)$ factor, as in the USSM [2]. In all cases one has to make sure that no supersymmetric minimum of the potential exists anywhere, otherwise supersymmetry would remain (or return to) conserved.

2) The Lagrangian density may appeal to a term proportional to the auxiliary (F or G) components of a singlet chiral superfield $S(x, \theta)$. In this case the superpotential

$$\mathcal{W} = \sigma S + \dots \quad (18)$$

includes a term linear in the singlet superfield S . One can then try to make the system of equations $\{F_i = G_i = 0\}$, i.e. $\partial \mathcal{W} / \partial \phi_i = 0$, without solution.

This first looks as an impossible task. Indeed with the superpotential

$$\mathcal{W} = \frac{\lambda_{ijk}}{3} \Phi_i \Phi_j \Phi_k + \frac{\mu_{ij}}{2} \Phi_i \Phi_j + \sigma_i \Phi_i, \quad (19)$$

taken for simplicity as a cubic function of the chiral superfields Φ_i for the theory to be renormalisable, the set of equations $\partial \mathcal{W} / \partial \phi_i = 0$ reads

$$\lambda_{ijk} \phi_j \phi_k + \mu_{ij} \phi_j + \sigma_i = 0. \quad (20)$$

With n equations for n complex field variables it is expected to have *almost always* solutions, for which supersymmetry is conserved.

Still it is possible choosing very carefully the set of interacting superfields and the superpotential \mathcal{W} for spontaneous supersymmetry breaking to occur. This appeals to a new symmetry called R symmetry [1]. R transformations, further discussed in the next subsection, act on (left-handed) chiral superfields according to

$$\Phi(x, \theta) \xrightarrow{R} e^{i R_\Phi \alpha} \Phi(x, \theta e^{-i \alpha}). \quad (21)$$

The superpotential must transform with $R = 2$ for the theory to be invariant under R . It is then be said to be “ R -symmetric”. This symmetry can be used to select and constrain appropriately the superpotential \mathcal{W} so that the set of equations (20) has no solution and the corresponding breaking of the supersymmetry is obtained *in a generic way*, not just for very specific values of the parameters [30, 31].

An interesting example is obtained with a R symmetric nMSSM-type superpotential [1], extended to a chiral triplet T . It involves, as in the MSSM without the μ term, the two doublets H_1 and H_2 with $R = 0$. They are coupled to a singlet S through a $\lambda H_2 H_1 S$ trilinear term as in the nMSSM, and similarly to a triplet T , both with $R = 2$. The corresponding $R = 2$ superpotential reads [30]

$$\mathcal{W} = \frac{1}{\sqrt{2}} H_2 (g \tau T - g' S) H_1 + \sigma S, \quad (22)$$

H_1 and H_2 having weak hypercharges $Y = -1$ and $+1$, respectively. This one is also, in addition, the superpotential for a $N = 2$ supersymmetric gauge theory (or “hypersymmetric” theory) [43] when the $SU(2) \times U(1)$ symmetry is made again local, with the superpotential couplings fixed in terms of the electroweak gauge couplings as in (22). Its two terms may be written as proportional to

$$H_2 \Phi H_1 \quad \text{and} \quad \text{Tr } \Phi, \quad (23)$$

with $\Phi = \frac{1}{2} (g \tau T - g' S)$. Or $\Phi = g \Lambda T - \frac{g'}{2} S$ where the chiral superfields T are in the adjoint representation of the gauge group, and the matrices Λ are relative to the hypermultiplet representation of the gauge group described by H_1 and H_2 .

For a non-abelian $N = 2$ theory the superpotential (22) reads

$$\mathcal{W} = g \sqrt{2} H_2 \Lambda T H_1, \quad (24)$$

leading to the $N = 4$ supersymmetric Yang-Mills theory when H_1 and H_2 are also taken in the adjoint representation of the gauge group [44]. The adjoint gauge superfield then interacts with 3 adjoint chiral ones, now denoted by S_1, S_2 and S_3 , coupled through the trilinear superpotential

$$\mathcal{W} = g \sqrt{2} f_{ijk} S_1^i S_2^j S_3^k. \quad (25)$$

But we shall return to this later.

Let us come back to the superpotential (22) for a non-gauged $SU(2) \times U(1)$ theory involving at the moment chiral superfields only, in view of generating a spontaneous breaking of the supersymmetry through F terms [30]. The conjugates of the 4 (complex) auxiliary components of the $R = 0$ superfields H_1, H_2 have $R = 2$, and vanish with the 4 components of t and s (also with $R = 2$). The conjugates of the 4 auxiliary components of the $R = 2$ superfields T and S have $R = 0$, and depend quadratically on the 4 components of h_1 and h_2 . One might naively expect that they should also, “generically”, be able to vanish simultaneously, so that supersymmetry would be conserved. Still this does not happen, as it would require

$$\begin{cases} \frac{\partial \mathcal{W}}{\partial t} = \frac{g}{\sqrt{2}} h_2 \tau h_1 = 0, \\ \frac{\partial \mathcal{W}}{\partial s} = \frac{g'}{\sqrt{2}} h_2 h_1 + \sigma = 0, \end{cases} \quad (26)$$

which are incompatible for $\sigma \neq 0$.

Supersymmetry is spontaneously broken, with a massless Goldstone spinor ζ_γ , taken as left-handed, carrying $R = 1$. It is described, together its $R = 2$ spin-0 partner, which remains classically massless, by a combination of the $R = 2$ chiral superfields.

This $R = 2$ chiral superfield is in this example the photon-like combination $\sin \theta T_3 + \cos \theta S$. It will be understood further by turning again the theory into a $N = 2$ supersymmetric one [43], with a $SU(2) \times U(1)$ gauge group spontaneously broken into $U(1)_{\text{QED}}$. The present Goldstone spinor ζ_γ associated with this F -breaking of the supersymmetry is described by the $R = 2$ chiral superfield $\sin \theta T_3 + \cos \theta S$. It then gets interpreted as the second photino field within $N = 2$, both photino fields λ_γ and ζ_γ , now related by a global $U(2)_R$ symmetry of $N = 2$, being the two Goldstone spinors of $N = 2$ supersymmetry.

D. On the role of R symmetry to allow for supersymmetry breaking through F terms

Without R symmetry, S^2 or S^3 terms would be allowed in the superpotential, and we would lose the benefit of having introduced a linear σS term, which may then be eliminated by a translation of S . Once σ is eliminated the potential has a vanishing minimum when all physical fields vanish, and supersymmetry is conserved. This shows the crucial role played by R symmetry to render possible a generic breaking of the supersymmetry through F terms [30].

This mechanism leads to a classically-massless $R = 2$ spin-0 field, superpartner of the $R = 1$ Goldstone spinor (goldstino). Both are described by a $R = 2$ chiral superfield. A translation of its $R = 2$ spin-0 component, if it had to be performed, would lead to a spontaneous breaking of R symmetry (or a quasi-spontaneous breaking if

R is anomalous). The imaginary part of this $R = 2$ field would then describe a massless R Goldstone boson or, for an anomalous symmetry, a classically-massless R -axion.

But is there a tighter connection between spontaneous supersymmetry breaking by this method, and the possible occurrence of spontaneous R -symmetry breaking through the v.e.v. of such a $R = 2$ scalar, superpartner of the $R = 1$ goldstino? The above example indicates that there is no need for R -symmetry to be spontaneously broken, to get spontaneous supersymmetry breaking. Conversely, for $\sigma = 0$ R symmetry may indeed be spontaneously broken, owing the $R = 2$ flat directions of the potential, with a massless R Goldstone boson, and a conserved supersymmetry. Thus spontaneous R -symmetry breaking is not a sufficient condition either, for spontaneous supersymmetry breaking to occur.

It thus appears that while spontaneous R -symmetry breaking may occur, *it is neither necessary nor sufficient* to lead to such a spontaneous breaking of the supersymmetry through F terms. What is indeed essential is the presence of R symmetry to restrict appropriately the superpotential as in (22) [30].

Furthermore, and in contrast with a current belief, spontaneous supersymmetry breaking occurs here, for $\sigma \neq 0$, in spite of having equal numbers of $R = 2$ and $R = 0$ superfields. There is thus no excess of $R = 1$ over $R = -1$ (left-handed) spinors, that would facilitate having a massless left-over $R = 1$ spinor that could become a Goldstone spinor. In fact with the same number of $R = 2$ and $R = 0$ superfields one might usually expect all spinors to acquire masses. Then there would be no candidate for a massless Goldstone spinor, and supersymmetry would have to remain conserved. Indeed the auxiliary components of the $R = 2$ superfields, which have $R = 0$, then depend on the same number of $R = 0$ physical fields, and might be expected to all vanish simultaneously.

Still this additional obstruction could be bypassed so as to render the system (26) of 4 equations for 4 variables generically without solution, and obtain spontaneous supersymmetry breaking [30]. This has been made possible, in particular, thanks to the spontaneously broken global $SU(2) \times U(1) \rightarrow U(1)$ symmetry generated by $\langle h_1 \rangle$ and $\langle h_2 \rangle$, leading to exactly massless $R = 0$ spin-0 Goldstone fields associated with $R = -1$ spinors. One of the latter balances a massless $R = 1$ spinor that is going to be the goldstino.

E. Unifying D - and F - breakings within $N = 2$, and going to $N = 4$ supersymmetry

Beyond that, when $SU(2) \times U(1)$ is gauged again as in the nMSSM, with the gauge superfields V and V' associated to the chiral triplet and singlet T and S , the theory based on the superpotential (22) acquires an enhanced

symmetry, namely extended $N = 2$ supersymmetry (or hypersymmetry), with H_1 and H_2 jointly describing a $N = 2$ hypermultiplet [43]. Of course no superpotential term proportional to S^2 , S^3 as in the general NMSSM, T^2 or ST^2 may be allowed here. Such terms, which would ruin the possibility of having a $N = 2$ supersymmetry, were already excluded by means of R -symmetry, which showed the way to extended supersymmetry, and subsequently extra dimensions.

The D - and F -breaking mechanisms then become equivalent, getting unified within $N = 2$ supersymmetry. Indeed the set of auxiliary components $\{-G, -F, D\}$ for a $N = 2$ gauge multiplet transform as the three components of a $SU(2)_R$ isotriplet within a $U(2)_R = [SU(2) \times U(1)]_R$ global symmetry group. The ξD term for a $U(1)$ gauge superfield can then be turned into a ξF term for its associated chiral superfield through a $SU(2)_R$ transformation turning the first supersymmetry generator into the second. This $N = 2$ supersymmetry breaking generates two massless Goldstone spinors, both with $R = 1$. A $SU(2) \times U(1)$, or more generally $G_{\text{non-abelian}} \times U(1)$ gauge group is then required if we intend to get a spontaneous breaking of the extended supersymmetry rather than just of the gauge symmetry [43].

For a non-abelian $N = 2$ gauge theory the superpotential reads $\mathcal{W} = g\sqrt{2} H_2 \Lambda T H_1$ as in (24), with T in the adjoint representation and the Λ representing the gauge group for the hypermultiplet described by H_1 and H_2 . A $N = 2$ supersymmetric theory with a massless matter hypermultiplet in the adjoint representation provides the $N = 4$ supersymmetric Yang-Mills theory [44]. The adjoint gauge superfield interacts with 3 adjoint chiral ones S_1, S_2 and S_3 coupled through the trilinear superpotential (25), $\mathcal{W} = g\sqrt{2} f_{ijk} S_1^i S_2^j S_3^k$. This provides the

$$\begin{aligned} & N = 4 \text{ supersymmetric Yang-Mills theory, with} \\ & (1 \text{ spin-1} + 4 \text{ spin-}\frac{1}{2} + 6 \text{ spin-0}) \text{ adjoint gauge fields.} \end{aligned} \tag{27}$$

The R symmetry acting chirally on the $N = 1$ supersymmetry generator (see later eq.(35)) gets promoted from $U(1)_R$ in $N = 1$ up to $SU(2)_R$ or $U(2)_R$ in $N = 2$, and $SU(4)_R \sim O(6)_R$ in $N = 4$ supersymmetry. This corresponds to the following chain

$$\begin{aligned} R\text{-parity} &\subset U(1)_R \subset \underbrace{SU(2)_R}_{N=2} \subset U(2)_R \\ &\subset \underbrace{SU(4)_R \sim O(6)_R}_{N=4}. \end{aligned} \tag{28}$$

The spontaneous breaking of the gauge symmetry in a $N = 2$ theory will be very useful, providing larger associations between massive spin-1 gauge bosons, spin- $\frac{1}{2}$ charginos and neutralinos and spin-0 BEH bosons [45], and leading us to a description of particle physics in a higher-dimensional space-time [46].

F. Origin of R symmetry, and of the extra $U(1)$ rotating h_1 and h_2

R symmetry originates from an earlier Q symmetry acting within a precursor of a supersymmetric theory including two spin-0 doublets (now called h_1 and h_2), and a Dirac spin- $\frac{1}{2}$ doublet, subsequently providing the corresponding higgsinos \tilde{h}_{1L} and \tilde{h}_{2L} [47]. The Q symmetry restricting both the possible form of the potential V and of the Yukawa couplings responsible for fermion masses was already a R -type symmetry. It acts according to

$$H_1 \xrightarrow{Q} e^{i\alpha} H_1(x, \theta e^{-i\alpha}), \quad H_2 \xrightarrow{Q} e^{i\alpha} H_2(x, \theta e^{-i\alpha}), \quad (29)$$

allowing for a $\mu H_2 H_1$ superpotential mass term for H_1 and H_2 . It was then turned into the R symmetry familiar to us today, defined as $R = Q U^{-1}$ (or equivalently $Q = R U$) and acting according to [1]

$$H_1 \xrightarrow{R} H_1(x, \theta e^{-i\alpha}), \quad H_2 \xrightarrow{R} H_2(x, \theta e^{-i\alpha}). \quad (30)$$

This R symmetry leaves h_1 and h_2 invariant so as to survive the electroweak breaking.

Here U denotes a $U(1)$ symmetry transformation commuting with supersymmetry, acting on the two electroweak doublets h_1 and h_2 according to

$$h_1 \xrightarrow{U} e^{i\alpha} h_1, \quad h_2 \xrightarrow{U} e^{i\alpha} h_2, \quad (31)$$

or in terms of superfields,

$$H_1 \xrightarrow{U} e^{i\alpha} H_1, \quad H_2 \xrightarrow{U} e^{i\alpha} H_2. \quad (32)$$

This definition was immediately extended in [1] to the extra nMSSM singlet S transforming according to

$$S \xrightarrow{U} e^{-2i\alpha} S. \quad (33)$$

The transformation (31) was first introduced as a way to constrain the potential in a two-doublet model by *allowing for independent phase transformations of h_1 and h_2* , jointly with the weak hypercharge $U(1)_Y$ (h_1 and h_2 having $Y = -1$ and $+1$) [47]. This does not lead to the appearance of an axion or axionlike particle as long as we are dealing with an *inert-doublet model*, keeping an unbroken symmetry combining a $U(1)$ transformation (31) with a $U(1)_Y$ transformation, under which

$$h_1 \rightarrow e^{2i\alpha} h_1, \quad h_2 \rightarrow h_2. \quad (34)$$

This residual $U(1)$ includes a Z_2 discrete symmetry under which the inert doublet h_1 changes sign, $h_1 \rightarrow -h_1$, and allows for a non-vanishing v.e.v. $\langle h_2 \rangle \neq 0$, which breaks spontaneously the electroweak symmetry. Such inert-doublet models can thus also provide, from the stability of the lightest component of h_1 , a possible dark matter candidate.

In supersymmetric extensions of the standard model, however, both h_1 and h_2 must acquire non-vanishing v.e.v.'s. A classically massless particle (A) would then appear in the spectrum as a consequence of the additional $U(1)$ symmetry (31,32), if this one remains indeed present. This particle is immediately apparent in the spectrum in the absence of the extra singlet superfield S (i.e. for $\lambda = 0$). Such a feature, considered as undesired, was *avoided from the beginning* by breaking explicitly the extra- $U(1)$ symmetry (31,32) through the introduction of the singlet S transforming as in (33). This singlet is coupled to H_1 and H_2 by a trilinear superpotential term $\lambda H_2 H_1 S$, invariant under the extra- $U(1)$.

The introduction of the linear term σS in the nMSSM superpotential $\lambda H_2 H_1 S + \sigma S$ breaks explicitly the extra- $U(1)$ symmetry (31-33), providing a mass $\lambda v/\sqrt{2}$ for the would-be “axion” A [1]. Its mass vanishes with λ , the extra- $U(1)$ symmetry with its associated Goldstone (or pseudo-Goldstone) boson A getting recovered for $\lambda = 0$. The same $U(1)$ transformation (31) acting on the two doublets h_1 and h_2 became useful later in a different context, to rotate away the CP -violating parameter θ of QCD [48]. The resulting presence of an axion A , after having escaped attention in [48], was pointed out in [49, 50].

But no such axion as been observed yet. This may be understood if the extra- $U(1)$ symmetry is broken at a high scale through a large v.e.v. $\langle s \rangle$ for a singlet transforming non-trivially under the extra $U(1)$, as in (33). We shall return to this in subsection IV B, when dealing with the interactions of a very light neutral spin-1 gauge boson Z' (or U) as may be present in the USSM, in which the extra- $U(1)$ symmetry (32,33) is gauged [2]. This light spin-1 boson would behave very much as the corresponding eaten-away axionlike pseudoscalar a , then mostly an electroweak singlet and interacting very weakly, thus largely “invisible” [51, 52].

G. Action of R symmetry

Let us now return to R symmetry. It enlarges the initial supersymmetry algebra (8) by introducing the new symmetry generator R corresponding to an abelian group $U(1)_R$. It acts chirally on the supersymmetry generator Q according to

$$Q \xrightarrow{R} e^{-\gamma_5 \alpha} Q, \quad (35)$$

or equivalently $Q_L \rightarrow e^{-i\alpha} Q_L$, transforming gauge and (left-handed) chiral superfields according to

$$\begin{cases} V(x, \theta, \bar{\theta}) \xrightarrow{R} V(x, \theta e^{-i\alpha}, \bar{\theta} e^{i\alpha}), \\ \Phi(x, \theta) \xrightarrow{R} e^{i R_\Phi \alpha} \Phi(x, \theta e^{-i\alpha}). \end{cases} \quad (36)$$

The spin-0 components $\phi = \Phi(x, 0)$ of chiral superfields transform with R quantum numbers R_Φ . Their associated spin- $\frac{1}{2}$ components $\tilde{\phi}_L$, proportional to $[Q_L, \phi]$

(or equivalently to the linear term in the expansion of Φ with respect to the Grassmann coordinate θ), have $R = R_\Phi - 1$. The R symmetry transformations (36) thus act on field components as

$$\begin{cases} V^\mu \xrightarrow{R} V^\mu, & \lambda \xrightarrow{R} e^{\gamma_5 \alpha} \lambda, \\ \phi \xrightarrow{R} e^{iR_\Phi \alpha} \phi, & \tilde{\phi}_L \xrightarrow{R} e^{i(R_\Phi - 1)\alpha} \tilde{\phi}_L, \end{cases} \quad (37)$$

λ denoting the Majorana gaugino fields associated with the gauge fields V^μ . The (complex) auxiliary components $(F + iG)/\sqrt{2}$ of the chiral superfields Φ transform with $R = R_\Phi - 2$, according to

$$\frac{F + iG}{\sqrt{2}} \xrightarrow{R} e^{i(R_\Phi - 2)\alpha} \frac{F + iG}{\sqrt{2}}. \quad (38)$$

The auxiliary components of $R = 2$ superfields are invariant under R . This was used in (18,22) to include a linear contribution σS within the superpotential \mathcal{W} of a R -symmetric theory, as in the nMSSM [1].

H. Constructing Dirac charginos and neutralinos with a conserved R symmetry

R symmetry (i.e. $U(1)_R$) allows in particular for R -invariant Yukawa couplings of gauginos to spin- $\frac{1}{2}$ and spin-0 fields described by chiral superfields, that may be expressed as

$$\mathcal{L}_Y = \sum_a (i) g_a \sqrt{2} \bar{\lambda}_{aR} \phi_i^\dagger (T_a)_{ij} \tilde{\phi}_{jL} + \text{h.c.}, \quad (39)$$

with

$$\lambda_{aR} \xrightarrow{R} e^{-i\alpha} \lambda_{aR}, \quad \phi_i^\dagger (T_a)_{ij} \tilde{\phi}_{jL} \xrightarrow{R} e^{-i\alpha} \phi_i^\dagger (T_a)_{ij} \tilde{\phi}_{jL}. \quad (40)$$

The phase factor ± 1 or $\pm i$ that may appear in front of the first term in (39) is convention-dependent and may be modified by a chiral redefinition of the gaugino fields λ_a , or a relative phase redefinition of ϕ_i and ϕ_{iL} .

This leads to the possibility of generating, through a spontaneous breaking of the gauge symmetry, R -invariant non-diagonal mass terms connecting gauginos with higgsinos, transforming as

$$\begin{cases} \text{gauginos } \lambda \xrightarrow{R} e^{\gamma_5 \alpha} \lambda, \\ \text{higgsinos } \psi \xrightarrow{R} e^{-\gamma_5 \alpha} \psi. \end{cases} \quad (41)$$

One thus gets Dirac spinors known as charginos and neutralinos – even if denominations like winos, zino, etc. could be more appropriate as we shall see. They may be expressed as [1]

$$\begin{cases} R = +1 \text{ Dirac ino} = \text{gaugino}_L + \text{higgsino}_R, \\ \text{or} \\ R = -1 \text{ Dirac ino} = \text{higgsino}_L + \text{gaugino}_R. \end{cases} \quad (42)$$

They have the same masses m_W , m_Z , etc. as the corresponding spin-1 gauge bosons, as long as supersymmetry is unbroken. This already hints at “gauge/BE-Higgs unification”, a crucial property that may be the prime motivation for supersymmetry [7, 8].

The introduction of direct gaugino (m_1, m_2) and higgsino (μ) mass terms then modifies these R -conserving chargino and neutralino mass matrices by including supersymmetry-breaking $\Delta R = \pm 2$ contributions. The μ parameter may be considered as “supersymmetric” as a $\mu H_2 H_1$ mass term may be included directly in the superpotential, or regenerated from the $\lambda H_2 H_1 S$ coupling through the translation of the $R = 2$ spin-0 component of the singlet S , leading to

$$\mu = \lambda \langle s \rangle. \quad (43)$$

Still the μ term generates a supersymmetry-breaking contribution to the mass matrices when the spin-0 doublets h_1 and h_2 acquire non-vanishing v.e.v.’s, by contributing to non-vanishing v.e.v.’s for the auxiliary components of H_1 and H_2 .

I. From R symmetry to R parity

R symmetry was introduced for reasons related with the triggering of the electroweak breaking induced by h_1 and h_2 , which must both acquire non-vanishing v.e.v.’s.. Otherwise we would stay with an unwanted massless chargino, even before thinking about introducing quarks and leptons and generating their masses. R symmetry was also introduced with the desire of defining a conserved quantum number R attributed to massless or massive Dirac spinors as in (41,42), with differences $\Delta R = \pm 1$ between fermions and bosons within the multiplets of supersymmetry.

This was done in a toy-model attempt at relating the photon with a “neutrino” carrying one unit of R , and the W^- with a light chargino that might have been an “electron” candidate (or even in 1976, at the time of the τ discovery, a τ candidate, with the fermionic partner of the photon as a ν_τ candidate). But the previous “neutrino”, called a gaugino in modern language, must in fact be considered as a new photonic neutrino within supersymmetric extensions of the standard model [2]. It was called the photino, with, similarly, the spin- $\frac{1}{2}$ partners of the gluons called the gluinos [32, 53], so that

$$\begin{cases} \text{photon} \xleftrightarrow{\text{SUSY}} \text{photino}, \\ \text{gluons} \xleftrightarrow{\text{SUSY}} \text{gluinos}. \end{cases} \quad (44)$$

The parity of the new quantum number R carried by the supersymmetry generator,

$$R_p = (-1)^R, \quad (45)$$

plays an important role. It distinguishes between ordinary particles, with $R_p = +1$, and superpartners, also

called *sparticles*, with $R_p = -1$, while allowing for the generation of masses for the Majorana spin- $\frac{3}{2}$ gravitino and spin- $\frac{1}{2}$ gluinos, which transform chirally under R symmetry [32, 53]. Their mass terms break explicitly the continuous R symmetry, reducing it to R -parity. This one may then be identified as [2, 3]

$$R_p = (-1)^R = (-1)^{2S} (-1)^{3B+L}. \quad (46)$$

As $R_p = (-1)^{2S}(-1)^{3(B-L)}$, its conservation follows from the conservation of $B - L$, even only modulo 2, ensuring the stability of the lightest supersymmetric particle, or LSP. This remains valid even in the presence of neutrino Majorana mass terms.

All superpartners are then expected to decay so as to ultimately provide, at the end of the decay chain, a stable LSP, usually taken to be a neutralino or a light gravitino [32], although other possibilities may also be considered. The neutralino, in particular, turns out to be a good candidate for the non-baryonic dark matter of the Universe.

Conversely, should R -parity necessarily be conserved? A non-conservation of R -parity, as in R_p -violating theories [54], requires B and/or L violations. It usually leads to severe difficulties with unobserved effects such as a much-too-fast proton decay mediated by squark exchanges, or too large neutrino masses, unless the corresponding products of R_p -violating couplings are taken sufficiently small. Also, if R -parity is no longer conserved, we generally lose the possibility of having a stable LSP as a candidate for the non-baryonic dark matter of the Universe.

IV. N/nMSSM AND MSSM SUPERPOTENTIALS AND POTENTIALS

A. Superpotentials

Let us precise the role of R symmetry in restricting adequately the superpotentials considered. The last component of the superpotential \mathcal{W} provides a contribution to the Lagrangian density invariant under supersymmetry, up to a derivative which does not contribute to the action integral. For the theory to be invariant under R its superpotential \mathcal{W} must transform according to

$$\mathcal{W}(x, \theta) \xrightarrow{R} e^{2i\alpha} \mathcal{W}(x, \theta e^{-i\alpha}), \quad (47)$$

so that its last component, which appears as the coefficient of the $\theta\theta$ term in its expansion and contributes to \mathcal{L} , be R -invariant.

A product of chiral superfields $\Pi\Phi_i$ transforms with $R = \sum R_{\Phi_i}$, and is allowed in the superpotential if and only if

$$\sum R_{\Phi_i} = 2. \quad (48)$$

The parameters λ_{ijk} , m_{ij} and σ_i in the superpotential

$$\mathcal{W} = \frac{\lambda_{ijk}}{3} \Phi_i \Phi_j \Phi_k + \frac{\mu_{ij}}{2} \Phi_i \Phi_j + \sigma_i \Phi_i \quad (49)$$

are required by R symmetry to vanish, unless the corresponding products of superfields verify $R_{\Phi_i} + R_{\Phi_j} + R_{\Phi_k} = 2$, $R_{\Phi_i} + R_{\Phi_j} = 2$, or $R_{\Phi_i} = 2$.

These restrictions from R symmetry are used to select the nMSSM superpotential for the two electroweak doublets H_1 and H_2 interacting with an extra singlet S through a trilinear superpotential coupling $\lambda H_2 H_1 S$ [1],

$$\mathcal{W}_{\text{nMSSM}} = S(\lambda H_2 H_1 + \sigma). \quad (50)$$

The terms involving quarks and leptons will be considered later [2]. This superpotential is obtained by imposing R symmetry on the general NMSSM superpotential, also including a $\mu H_2 H_1$ mass term as well as mass and self-interaction terms for S ,

$$\mathcal{W}_{\text{NMSSM}} = S(\lambda H_2 H_1 + \sigma) + \mu H_2 H_1 + \frac{\kappa}{3} S^3 + \frac{\mu_S}{2} S^2. \quad (51)$$

H_1 and H_2 transform as in (30) so that R symmetry can survive the electroweak breaking, extended to S according to

$$H_{1,2} \xrightarrow{R} H_{1,2}(x, \theta e^{-i\alpha}), \quad S \xrightarrow{R} e^{2i\alpha} S(x, \theta e^{-i\alpha}). \quad (52)$$

Both $\lambda H_2 H_1 S$ and σS are allowed by R in the nMSSM superpotential (50). The other NMSSM terms in (51), proportional to $H_2 H_1$, S^2 and S^3 , are excluded.

Another way to restrict the general NMSSM superpotential (51) into the nMSSM one (50) is to ask for \mathcal{W} to be invariant under the extra- $U(1)$ symmetry (32,33) [1],

$$H_i \xrightarrow{U} e^{i\alpha} H_i, \quad S \xrightarrow{U} e^{-2i\alpha} S, \quad (53)$$

simply broken by the dimension-2 linear term σS , thus automatically avoiding a classically massless spin-0 “axion”, before this notion was even put into light. This extra- $U(1)$ symmetry also excludes NMSSM self-interaction and mass terms proportional to S^3 and S^2 in the superpotential, as well as $\mu H_2 H_1$. The latter may still be subsequently regenerated through a translation of S as in (43).

Incidentally, the μ parameter, coefficient of the $\mu H_2 H_1$ superpotential mass term in the MSSM, is “supersymmetric” (in the sense that $\mu H_2 H_1$ may be present in the superpotential) but comes in violation of both the R -symmetry (52) and the extra- $U(1)$ symmetry (53). It may thus remain naturally small or of moderate size, as compared to very large mass scales like the grand-unification or the Planck scales.

A special version of the above general NMSSM superpotential (51) involves trilinear terms only in the superpotential [55, 56], with

$$\mathcal{W}_{\text{NMSSM}} = \lambda H_2 H_1 S + \frac{\kappa}{3} S^3, \quad (54)$$

λ and κ being dimensionless. Most of its interesting properties rely on the same trilinear $\lambda H_2 H_1 S$ coupling as in the nMSSM. In the limit $\kappa \rightarrow 0$, both the $U(1)_R$ (52) and the extra- $U(1)$ (53) would be restored. The latter being broken by $\langle h_1 \rangle$ and $\langle h_2 \rangle$ (and $\langle s \rangle$ if also present) a classically massless axionlike boson (a) would then reappear in this limit, that was precedently avoided in the nMSSM by the linear σS term (and in the above version of the NMSSM by $\frac{\kappa}{3} S^3$). Such a particle, which has not been observed, may also acquire a mass, possibly small, through the soft supersymmetry-breaking terms breaking explicitly the extra- $U(1)$ symmetry.

B. The USSM, with a new neutral gauge boson

Another option is to gauge the above extra- $U(1)$ symmetry (53), assuming the corresponding anomalies appropriately cancelled, usually through the introduction of extra fermion fields. These may involve, for example, mirror fermions, or exotic fermions as would be present in an $E(6)$ theory. The would-be (axionlike) Goldstone boson (a) is then “eaten away” when the additional neutral gauge boson Z' acquires a mass. This leads to the USSM, with the trilinear superpotential

$$\mathcal{W}_{\text{USSM}} = \lambda H_2 H_1 S, \quad (55)$$

the theory being at this stage invariant under both the R symmetry (52) and the extra- $U(1)$ symmetry (53), now promoted to a local gauge symmetry [2].

The gauging of an additional $U(1)$, possibly appearing as a subgroup of a non-abelian grand-unification group like $E(6)$, with (anti)quark and (anti)lepton chiral superfields transforming axially according to

$$(L, Q; \bar{E}, \bar{D}, \bar{U}) \xrightarrow{U} e^{-\frac{i\alpha}{2}} (L, Q; \bar{E}, \bar{D}, \bar{U}), \quad (56)$$

requires a new spin-1 gauge boson Z' . More generally the extra- $U(1)$ symmetry generator to be gauged may involve a linear combination of the axial $U(1)$ quantum number defined from (53,56) as

$$\begin{cases} F_A(L, Q; \bar{E}, \bar{D}, \bar{U}) = -\frac{1}{2}, \\ F_A(H_1, H_2) = 1, \quad F_A(S) = -2. \end{cases} \quad (57)$$

with the weak hypercharge Y and the B and L (or $B-L$) quantum numbers. A large v.e.v. for an extra singlet like s , already present in the theory and transforming as in (53), $s \rightarrow e^{-2i\alpha} s$, may make the new gauge boson much heavier than the W^\pm and Z , giving it a large mass \gtrsim TeV scale [51]. But no new heavy boson corresponding to an enlargement of the gauge group has been discovered yet.

C. A new light gauge boson U ? or a light pseudoscalar a ?

There is also another interesting possibility. An additional $U(1)$ factor in the gauge group, if not embedded

within a grand-unification group like $O(10)$ or $E(6)$, …, would have its own gauge coupling constant g'' , next to g and g' . This one may be much smaller than g and g' , in which case the mass of the new neutral gauge boson may well be small. This Z' , also called a U boson, would then have, for its longitudinal polarisation state, effective interactions fixed by $g'' k^\mu / m_U$. It would behave very much as the “eaten-away” Goldstone boson a , acquiring effective axionlike pseudoscalar couplings to quarks and leptons recovered from its axial couplings f_A (proportional to g''), as [52]

$$f_p = f_A \frac{2m_{l,q}}{m_U}. \quad (58)$$

This is very similar to the situation for a massive but light spin- $\frac{3}{2}$ gravitino, with a very small gravitational coupling $\kappa = \sqrt{8\pi G_N} \simeq 4 \cdot 10^{-19} \text{ GeV}^{-1}$, and a small mass

$$m_{3/2} = \frac{\kappa d}{\sqrt{6}} = \frac{\kappa F}{\sqrt{3}}. \quad (59)$$

F , or \sqrt{F} , is usually referred to as the supersymmetry-breaking scale parameter. Such a light gravitino would have its $\pm 1/2$ polarisation states interacting proportionally to $\kappa k^\mu / m_{3/2}$, or k^μ / F . It would still behave very much as the “eaten-away” spin- $\frac{1}{2}$ goldstino, according to the “equivalence theorem” of supersymmetry. The strength of its interactions then depends on the scale at which supersymmetry is spontaneously broken, getting very small if the supersymmetry-breaking scale (\sqrt{d} or \sqrt{F}) is large enough [32].

Let us return to a light spin-1 U boson. As it would behave very much like the corresponding equivalent Goldstone boson [52], it would certainly be excluded if it could be produced, most notably in the radiative decays of the ψ and the Υ , much like a standard axion (A). Fortunately the singlet s already present in these theories, transforming under U as in (53) according to $s \rightarrow e^{-2i\alpha} s$, may acquire a large v.e.v., significantly above the weak scale. The extra- $U(1)$ symmetry is then broken “at a large scale” F_U , where the mass $m_U \propto g'' F_U$ may still be small when the extra $U(1)$ is gauged with a very small coupling. The corresponding particle (either the very light spin-1 U boson or its “equivalent” spin-0 pseudoscalar a) is then coupled effectively very weakly, proportionally to g'' / m_U , or $1/F_U$ [51, 52]. This pseudoscalar a is mostly an electroweak singlet, largely inert.

Dealing with a spin-0 particle this also provided, as a by-product, a very early realization of the “invisible axion” mechanism that became popular later, in which the “invisible axion” is mostly an electroweak singlet [51]. Furthermore the doublet and singlet $U(1)$ quantum numbers are here appropriate to the supersymmetry framework, with an invariant $\lambda H_2 H_1 S$ trilinear coupling, resulting in the $U(1)$ quantum numbers +1 for h_1 and h_2 , $-1/2$ and $+1/2$ for left-handed and right-handed quarks and leptons, and -2 for the extra singlet s as in (57).

In a similar way a light spin-1 gauge boson U , interacting very much as the eaten-away Goldstone boson a i.e. as an “invisible axion” (except for the $\gamma\gamma$ coupling of the latter), also becomes largely “invisible” if the extra- $U(1)$ symmetry is broken at a sufficiently high scale. But the hunt for such a light spin-1 U boson is another story [57].

D. N/nMSSM and MSSM potentials

The nMSSM superpotential (50) leads to the potential [1]

$$\begin{aligned} V_{\text{nMSSM}} = & \frac{g^2 + g'^2}{8} (h_1^\dagger h_1 - h_2^\dagger h_2)^2 + \frac{g^2}{2} |h_1^\dagger h_2|^2 \\ & + \frac{\xi g'}{2} (h_1^\dagger h_1 - h_2^\dagger h_2) + \frac{\xi^2}{2} \\ & + |\lambda h_2 h_1 + \sigma|^2 + \lambda^2 |s|^2 (|h_1|^2 + |h_2|^2). \end{aligned} \quad (60)$$

The D -term contributions take into account an abelian $-\xi D'$ term in \mathcal{L} (this sign choice, different from the usual one in (15), being made to have $\xi > 0$ for $v_2 > v_1$ i.e. $\tan \beta > 1$).

This also applies to the general NMSSM through the replacements

$$\begin{aligned} \sigma S \rightarrow f(S) = & \frac{\kappa}{3} S^3 + \frac{\mu_S}{2} S^2 + \sigma S, \quad \lambda S \rightarrow \mu + \lambda S, \\ \sigma \rightarrow & \frac{df(s)}{ds} = \kappa s^2 + \mu_S s + \sigma, \quad \lambda s \rightarrow \mu + \lambda s, \end{aligned} \quad (61)$$

in the superpotential and potential, respectively, leading to

$$\begin{aligned} V_{\text{NMSSM}} = & \frac{g^2 + g'^2}{8} (h_1^\dagger h_1 - h_2^\dagger h_2)^2 + \frac{g^2}{2} |h_1^\dagger h_2|^2 \\ & + \frac{\xi g'}{2} (h_1^\dagger h_1 - h_2^\dagger h_2) + \frac{\xi^2}{2} \\ & + |\lambda h_2 h_1 + \kappa s^2 + \mu_S s + \sigma|^2 + |\mu + \lambda s|^2 (|h_1|^2 + |h_2|^2). \end{aligned} \quad (62)$$

The translation (43) of the singlet S restores the (N)MSSM mass term $\mu H_2 H_1$ from the nMSSM superpotential (50). Furthermore in the $\lambda \rightarrow 0, \sigma \rightarrow \infty$ limit, with $\lambda\sigma$ fixed, for which S decouples, we recover the MSSM potential, in the conceptually-interesting situation of a MSSM potential with dimension-2 *soft-breaking terms generated from a supersymmetric Lagrangian density*. It reads (up to a very large or infinite constant term, irrelevant at the moment)

$$\begin{aligned} V_{\text{MSSM}} = & \frac{g^2 + g'^2}{8} (h_1^\dagger h_1 - h_2^\dagger h_2)^2 + \frac{g^2}{2} |h_1^\dagger h_2|^2 \\ & (\mu^2 + \frac{\xi g'}{2}) h_1^\dagger h_1 + (\mu^2 - \frac{\xi g'}{2}) h_2^\dagger h_2 + 2 \lambda \sigma \Re h_2 h_1. \end{aligned} \quad (63)$$

The last term, $\propto \Re h_2 h_1$, forces h_1 as well as h_2 to acquire a non-vanishing v.e.v.. But this does not lead to an unwanted classically-massless axion or axionlike pseudoscalar A , as this term $\propto \Re h_2 h_1$ breaks explicitly the extra- $U(1)$ symmetry (31,32), $h_1 \rightarrow e^{i\alpha} h_1$, $h_2 \rightarrow e^{i\alpha} h_2$.

If the (extremely weak) interactions of the singlet S were reconsidered again, with an extremely small coupling λ , the vacuum state corresponding to (63), which then has an extremely large energy density $\simeq \sigma^2 \propto 1/\lambda^2$, would be destabilized, but still staying effectively quasi-metastable.

These expressions of the N/nMSSM and MSSM potentials illustrate how spin-0 interactions may now be viewed as *part of the electroweak gauge interactions*, with their quartic couplings fixed by

$$\frac{g^2 + g'^2}{8} \quad \text{and} \quad \frac{g^2}{2}. \quad (64)$$

They lead to a spontaneous breaking of $SU(2) \times U(1)$ into $U(1)_{\text{QED}}$, with non-vanishing v.e.v.’s for both h_1 and h_2 ,

$$\langle h_1 \rangle = \begin{pmatrix} \frac{v_1}{\sqrt{2}} \\ 0 \end{pmatrix}, \quad \langle h_2 \rangle = \begin{pmatrix} 0 \\ \frac{v_2}{\sqrt{2}} \end{pmatrix}, \quad (65)$$

where $v_1 = v \cos \beta$, $v_2 = v \sin \beta$ with $v \simeq 246$ GeV. This also leads us in the direction of gauge-Higgs unification already alluded to in (6) [7, 8, 42], discussed in the next Section.

The nMSSM potential (60), in particular, forces v_1 and v_2 to verify at this stage (before the introduction of extra terms breaking supersymmetry explicitly) $\sigma - \frac{1}{2} \lambda v_1 v_2 = 0$, which ensures the vanishing of the F terms in the potential. Minimizing the D terms leads (without a μ term yet) to $\langle D_Z \rangle = 0$ with $\langle D_\gamma \rangle \neq 0$ so that the photino is the Goldstone spinor, then fixing, in the absence of other soft-breaking terms,

$$m_Z^2 (-\cos 2\beta) = \xi g', \quad (66)$$

i.e. $\sqrt{\xi} \simeq m_Z / \sqrt{g'} \simeq 155$ GeV, in the large $\tan \beta$ limit [8].

The structure of the nMSSM superpotential (50) (and resulting potential as in (60)) is useful in many circumstances, and most notably to trigger gauge symmetry breaking by rendering the gauge-symmetric vacuum state unstable. It also leads to inflationary potentials useful in the description of the very early Universe, with an initial energy density such as $\sigma^2 + \xi^2/2$, providing the necessary fuel for inflation. Additional soft-breaking terms, of dimension ≤ 3 [58], possibly induced from supergravity [59–64], may also be added to the (N/n)MSSM potentials (60,62,63).

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V. GAUGE/BE-HIGGS UNIFICATION IN THE (N/n)MSSM

A. An index Δ for counting massless chiral spinors

These theories make use of gauge superfields, describing (left-handed) gaugino fields λ_L carrying $R = 1$, and $R = 2$ or 0 chiral superfields describing (left-handed) chiral spinors with $R = 1$ and -1 respectively, as seen from (37,42). With n_g gauge superfields and n_2 and n_0 chiral superfields with $R = 2$ or 0 , we get $n_g + n_2$ and n_0 (left-handed) spinors with $R = 1$ and -1 , respectively. The former are in excess by the difference

$$\Delta = \underbrace{n_g + n_2}_{R=1 \text{ spinors}} - \underbrace{n_0}_{R=-1 \text{ spinors}} . \quad (67)$$

$R = 1$ and -1 left-handed spinors may combine as in (42), in a way compatible with R symmetry, into massive Dirac spinors carrying $R = \pm 1$. For $\Delta \geq 0$, Δ chiral spinors with $R = 1$, at least, must remain massless if R symmetry is conserved, as staying unpaired with $R = -1$ counterparts.

With the nMSSM superpotential (50) $n_g = 4$ for $SU(2) \times U(1)$, $n_0 = 4$ for H_1 and H_2 and $n_2 = 1$ for S , so that $\Delta = 1$. One left-handed spinor with $R = 1$, neutral, must remain massless, which is here the photino. This one becomes the Goldstone spinor when supersymmetry is spontaneously broken. An early version of the USSM, with one additional extra- $U(1)$ gaugino, had $\Delta = 5 + 1 - 4 = 2$, leading to two massless $R = 1$ spinors, with a goldstino (eaten-away by the spin- $\frac{3}{2}$ gravitino) different from the photino, superpartners ultimately decaying into gravitinos or photinos carrying away missing energy-momentum [2, 32].

B. The goldstino must transform with $R = 1$

The (left-handed) massless Goldstone spinor λ_g associated with spontaneous supersymmetry breaking must have $R = 1$ in a R symmetric theory, as follows from the R transformation properties (35) of the supersymmetry generator and vector-spinor current, such that

$$J_\alpha^\mu \xrightarrow{R} e^{-\gamma_5 \alpha} J_\alpha^\mu . \quad (68)$$

For spontaneously broken supersymmetry the vector-spinor current may be expressed as

$$J_\alpha^\mu = d \gamma^\mu \gamma_5 \lambda_g + \dots , \quad (69)$$

where $d/\sqrt{2} = F$ is the supersymmetry-breaking scale parameter which determines the gravitino mass $m_{3/2} = \kappa d/\sqrt{6} = \kappa F/\sqrt{3}$ in (59) [32]. The Goldstone spinor must transform according to $\lambda_g \xrightarrow{R} e^{\gamma_5 \alpha} \lambda_g$, or equivalently

$$\lambda_{gL} \xrightarrow{R} e^{i\alpha} \lambda_{gL} , \quad (70)$$

i.e. it should transform with $R = 1$.

It should be either a gaugino as in pure D -breaking [1, 29, 42], or a spin- $\frac{1}{2}$ fermion field described by a $R = 2$ chiral superfield as in F -breaking [30, 31], or a mixing of both as in [2, 29]. In the nMSSM at the present stage, with an unbroken R symmetry and without any addition of soft supersymmetry-breaking terms yet, the massless goldstino field, with $R = 1$, coincides with the photino field, supersymmetry remaining unbroken within neutral multiplets. This degeneracy gets broken later through terms breaking explicitly (although softly) the supersymmetry [27, 58], possibly obtained from gravity-induced supersymmetry breaking [59–64]. Still the R -symmetric nMSSM considered at the present stage is essential in the understanding of the gauge/BE-Higgs unification and of the resulting mass spectrum for the various versions of the MSSM or N/nMSSM, as we shall see.

C. $U(1)_R$ symmetric nMSSM mass spectrum

With the gauge and chiral superfields transforming under the continuous R symmetry ($U(1)_R$) according to

$$\begin{cases} V_a & \xrightarrow{R} V_a(x, \theta e^{-i\alpha}, \bar{\theta} e^{i\alpha}) , \\ H_{1,2} & \xrightarrow{R} H_{1,2}(x, \theta e^{-i\alpha}) , \\ S & \xrightarrow{R} e^{2i\alpha} S(x, \theta e^{-i\alpha}) , \end{cases} \quad (71)$$

R symmetry leads to 1 chiral spinor remaining massless, with 4 massive Dirac ones. The R -symmetric and quasi-supersymmetric fermion spectrum for the nMSSM is at this stage, with $gv_1/\sqrt{2} = m_W \sqrt{2} \cos \beta$, $gv_2/\sqrt{2} = m_W \sqrt{2} \sin \beta$ [1]:

$\begin{cases} 1 \text{ massless photino:} & m = 0 , \\ 2 \text{ Dirac winos:} & m = \begin{cases} m_W \sqrt{2} \cos \beta , \\ m_W \sqrt{2} \sin \beta , \end{cases} \\ 1 \text{ Dirac zino:} & m_Z = \sqrt{g^2 + g'^2} v/2 , \\ 1 \text{ Dirac neutralino:} & m = \lambda v/\sqrt{2} , \end{cases}$

(72)

all spinors carrying $R = \pm 1$, in agreement with their expressions from gaugino and higgsino fields as in (42). The corresponding 5×5 neutralino mass matrix expressed in a gaugino-higgsino basis will be given later in subsection VID.

The charged and neutral spin-0 masses, obtained from the potential (60), are

$$\begin{cases} m_{H^\pm} = m_W , & m_z = m_Z , \\ m (4 \text{ neutral spin-0 bosons}) = \frac{\lambda v}{\sqrt{2}} . \end{cases} \quad (73)$$

All neutral spin-0 bosons have the same mass m_Z or $\lambda v/\sqrt{2}$ as their fermionic partners in (72), thanks to

the unbroken supersymmetry in the neutral sector, the photino being here the Goldstone spinor. Two of the four neutral bosons of mass $\lambda v/\sqrt{2}$ are described by the singlet superfield S , with $R = 2$. The two others are described by the $R = 0$ superfield

$$H_A = H_1^0 \sin \beta + H_2^0 \cos \beta. \quad (74)$$

As $\langle h_1^0 \rangle = v \cos \beta / \sqrt{2}$, $\langle h_2^0 \rangle = v \sin \beta / \sqrt{2}$, H_A acquires the mass $\lambda v / \sqrt{2}$ from the $\lambda H_2 H_1 S$ superpotential term, by combining with the singlet S according to

$$\lambda H_2 H_1 S = -\frac{\lambda v}{\sqrt{2}} (H_1^0 \sin \beta + H_2^0 \cos \beta) S + \dots, \quad (75)$$

in a way compatible with R symmetry (with $H_2 H_1 = -H_1 H_2 = H_2^+ H_1^- - H_2^0 H_1^0$).

All four scalars would return to massless for $\lambda \rightarrow 0$, for which S decouples, H_A returning to massless. Indeed in the $\lambda \rightarrow 0$ limit one recovers at the classical level a spontaneously broken extra- $U(1)$ acting as in (31,32), generating a classically-massless axion or axionlike particle. This one, which has here the mass $m_A = \lambda v / \sqrt{2}$, is described by the imaginary part of the spin-0 component of H_A ,

$$A = \sqrt{2} \operatorname{Im} (h_1^0 \sin \beta + h_2^0 \cos \beta). \quad (76)$$

D. Gauge/BE-Higgs unification

Relating gauge and BEH bosons, in spite of different electroweak properties

The superfield orthogonal to H_A in (74) is

$$H_z = -H_1^0 \cos \beta + H_2^0 \sin \beta. \quad (77)$$

The imaginary part of its spin-0 component,

$$z_g = \sqrt{2} \operatorname{Im} (-h_1^0 \cos \beta + h_2^0 \sin \beta), \quad (78)$$

orthogonal to the A field in (76), describes the would-be Goldstone boson eaten away by the Z . Indeed this Goldstone field z_g originates from the imaginary part of SM-like combination $\varphi_{\text{sm}}^\circ = h_1^0 \cos \beta + h_2^{0*} \sin \beta$ responsible for the electroweak breaking, with $\langle \varphi_{\text{sm}}^\circ \rangle = v / \sqrt{2}$.

The real part of the spin-0 component of H_z is

$$z = \sqrt{2} \operatorname{Re} (-h_1^0 \cos \beta + h_2^0 \sin \beta). \quad (79)$$

The signs are chosen for convenience so that $H_z \rightarrow H_2^0$ and $z \rightarrow \sqrt{2} \operatorname{Re} h_2^0$ in the large $\tan \beta$ limit. This field, suitably translated so that $\langle z \rangle = 0$, describes in this formalism the spin-0 boson partner of the Z within a massive gauge multiplet of supersymmetry. Its mass is obtained from the $D_Z^2/2$ contribution to the potential, as expressed in the nMSSM superpotential (60), with [8]

$$\begin{aligned} D_Z &= \frac{\sqrt{(g^2 + g'^2)} v}{2} [\sqrt{2} \operatorname{Re} (-h_1^0 \cos \beta + h_2^0 \sin \beta)] + \dots \\ &= m_Z z + \dots, \end{aligned} \quad (80)$$

and

$$\frac{1}{2} D_Z^2 = \frac{1}{2} m_Z^2 z^2 + \dots. \quad (81)$$

One thus has

$$m_z = m_Z \simeq 91 \text{ GeV}/c^2$$

(up to supersymmetry-breaking mass and mixing effects),

independently of the value of $\tan \beta$, in agreement with the unbroken supersymmetry in the neutral sector [1].

More precisely when the BEH mechanism operates within a supersymmetric theory, it provides *massive gauge multiplets* [42]. Each of them describes a massive spin-1 gauge boson, two spin- $\frac{1}{2}$ inos constructed from gaugino and higgsino components as in (42), and a spin-0 BEH boson associated with the spontaneous breaking of the gauge symmetry. We get systematic associations between massive gauge bosons and spin-0 BEH bosons, *a quite non-trivial feature owing to their different gauge symmetry properties*, and very different couplings to quarks and leptons [7, 8, 65].

We have in particular the association

$$Z \xleftrightarrow{\text{SUSY}} 2 \text{ Majorana zinos} \xleftrightarrow{\text{SUSY}} \text{spin-0 BEH boson.}$$

Independently of $\tan \beta$, and of λ in the presence of the N/nMSSM singlet S , the neutral spin-0 boson described by the z field in (79) becomes the spin-0 partner of the Z within a massive multiplet of supersymmetry. It has the same mass m_Z as long as supersymmetry is unbroken in this sector, in agreement with (6,72,73,81) [1, 7, 8].

This also applies to the W^\pm , according to

$$W^\pm \xleftrightarrow{\text{SUSY}} 2 \text{ Dirac winos} \xleftrightarrow{\text{SUSY}} \text{spin-0 boson } H^\pm.$$

The W^\pm is associated with two Dirac winos (usually known as charginos), obtained as in (42) with masses given in (72), and a charged spin-0 boson H^\pm (or w^\pm), with

$$w^\pm \equiv H^\pm = \sin \beta h_1^\pm + \cos \beta h_2^\pm, \quad (85)$$

approaching h_1^\pm at large $\tan \beta$. This one was originally called w^\pm in [1] to emphasize its relation with the W^\pm , leading in a model-independent way to

$$m_{H^\pm} = m_{W^\pm} \simeq 80 \text{ GeV}/c^2$$

(up to supersymmetry-breaking effects).

We shall see later how these mass equalities get modified in the presence of supersymmetry-breaking effects, in the MSSM and N/nMSSM.

As seen on (83,84) the supersymmetry generator Q has become able to relate bosons and fermions with different electroweak gauge symmetry properties, a quite non-trivial feature, in contrast with the usual belief. This makes supersymmetry a very powerful symmetry, much beyond the simple replication of degrees of freedom by associating bosons and fermions with the same gauge symmetry properties.

While massive gauge bosons and spin-0 BEH bosons have different symmetry properties for the electroweak gauge group, that is spontaneously broken, they do have the same properties for the $SU(3)_{\text{QCD}} \times U(1)_{\text{QED}}$ symmetries, that remain unbroken.

When supersymmetry is broken the lightest neutral spin-0 boson should presumably be identified with the 125 GeV boson found at CERN [5, 6] (unless a lighter one has escaped attention). This one may well correspond to the above z field (approaching $\sqrt{2} \Re h_2^0$ in the large $\tan \beta$ limit), up to a mixing angle, possibly small, induced by supersymmetry breaking.

However, the non-observation, at this stage, of a charged spin-0 BEH boson H^\pm seems to indicate (unless such a boson is found, with a moderate mass) that the effects of supersymmetry breaking are more important in the W^\pm than in the Z multiplet. This may be understood from the possible form of the supersymmetry-breaking terms.

E. Describing spin-0 BEH bosons by massive gauge superfields

This association between the spin-1 W^\pm and Z and the spin-0 H^\pm (also called w^\pm) and z can be made explicit in a different superfield formulation. Spin-0 BEH bosons will now be described by the spin-0 components of massive gauge superfields [7, 8, 42], after all components of the superfields H_1^- , H_2^+ and H_z in (77), then considered as chiral Goldstone superfields, get completely gauged away through the generalized gauge choices

$$H_1^- \equiv H_2^+ \equiv 0, \quad H_z \equiv \langle H_z \rangle = -\frac{v}{\sqrt{2}} \cos 2\beta. \quad (87)$$

In this new picture these spin-0 bosons get described, in a manifestly supersymmetric formulation, by the lowest (C) spin-0 components of massive Z and W^\pm superfields, expanded as $Z(x, \theta, \bar{\theta}) = C_Z + \dots - \theta \sigma_\mu \bar{\theta} Z^\mu + \dots$, $W^\pm(x, \theta, \bar{\theta}) = C_W^\pm + \dots - \theta \sigma_\mu \bar{\theta} W^{\mu\pm} + \dots$. Their spin-0 C components now describe, through non-polynomial field transformations linearized as $z = -m_Z C_Z + \dots$, $w^\pm = m_W C_W^\pm + \dots$, the same spin-0 fields z and w^\pm as in the usual formalism (with signs depending on previous choices for the definitions of z and w^\pm).

We thus have

$$\begin{cases} Z(x, \theta, \bar{\theta}) = (\frac{-z}{m_Z} + \dots) + \dots - \theta \sigma_\mu \bar{\theta} Z^\mu + \dots, \\ W^\pm(x, \theta, \bar{\theta}) = (\frac{w^\pm}{m_W} + \dots) + \dots - \theta \sigma_\mu \bar{\theta} W^{\mu\pm} + \dots. \end{cases} \quad (88)$$

The spin-0 components of massive gauge superfields now describe spin-0 BEH bosons! Their subcanonical (χ) spin- $\frac{1}{2}$ components, instead of being gauged-away as usual, now also correspond to physical degrees of freedom describing the spin- $\frac{1}{2}$ fields usually known as higgsinos.

Supersymmetry transformations act in a linear way on the components $(C, \chi, M, N, V^\mu, \lambda, D)$ of a massive gauge superfield $V(x, \theta, \bar{\theta})$, including auxiliary as well as physical components. But they act in a more complicated way when they are formulated in terms of the usual canonically-normalized spin-0 BEH and spin- $\frac{1}{2}$ higgsino fields, in particular as their expressions involve the dimensionless C components in a non-polynomial way.

VI. (N/n)MSSM MASS SPECTRA

with gauge/BE-Higgs unification

A. Spin-0 masses in the MSSM

The non-observation, at this stage, of a charged spin-0 BEH boson seems to indicate that the effects of supersymmetry breaking should be more important in the W^\pm than in the Z multiplet. This may be an effect of a significant supersymmetry-breaking term, possibly generated spontaneously from the decoupling limit of an extra singlet as indicated in (63), or from soft gravity-induced terms.

Let us define

$$\begin{cases} \varphi_{\text{sm}} = h_1 \cos \beta + h_2^c \sin \beta, \\ \varphi_{\text{in}} = h_1 \sin \beta - h_2^c \cos \beta, \end{cases} \quad (89)$$

so that φ_{sm} appears as a SM-like doublet responsible for the electroweak breaking and φ_{in} as an “inert doublet”, with $\langle \varphi_{\text{sm}} \rangle = v/\sqrt{2}$, $\langle \varphi_{\text{in}} \rangle = 0$. Viewing for convenience β as a fixed parameter unaffected by supersymmetry-breaking terms, these terms may be viewed as providing a mass term for the “inert” doublet φ_{in} , without modifying the vacuum state defined by $\langle h_1 \rangle$ and $\langle h_2 \rangle$.

One has, using (76) and (85),

$$\begin{aligned} |\varphi_{\text{in}}|^2 &= |h_1 \sin \beta - h_2^c \cos \beta|^2 \\ &= |H^+|^2 + \frac{1}{2} A^2 + \frac{1}{2} |\sqrt{2} \Re(h_1^0 \sin \beta - h_2^0 \cos \beta)|^2. \end{aligned} \quad (90)$$

Furthermore, if these dimension-2 supersymmetry-breaking terms expressed as $m_A^2 |\varphi_{\text{in}}|^2$ were generated as in

TABLE II: Minimal content of the Supersymmetric Standard Model (MSSM). Neutral gauginos and higgsinos mix into a photino, two zinos and a higgsino, further mixed into four neutralinos. Ordinary particles, including additional BEH bosons, in blue, have R -parity +1. Their superpartners, in red, have R -parity -1. The N/nMSSM includes an extra singlet with a trilinear $\lambda H_2 H_1 S$ superpotential coupling, describing a singlino and two additional neutral spin-0 bosons. The USSM also includes an extra neutral gauge boson Z' (or U) and its associated gaugino.

Spin 1	Spin 1/2	Spin 0
gluons photon	gluinos \tilde{g} photino $\tilde{\gamma}$	
W^\pm Z	winos $\widetilde{W}_{1,2}^\pm$ zinos $\widetilde{Z}_{1,2}$ higgsino \tilde{h}	H^\pm h H, A
	leptons l quarks q	sleptons \tilde{l} squarks \tilde{q}

(63) from a decoupling limit of the singlet S we would have

$$m_A^2 = 2\mu^2. \quad (91)$$

Or in a more general way, allowing for extra soft-breaking contributions for h_1 and h_2 ,

$$m_A^2 = 2\mu^2 + \Delta m^2(h_1) + \Delta m^2(h_2). \quad (92)$$

The mass term for φ_{in} provides equal contributions to m_A^2 and $m_{H^\pm}^2$, and leads to a further mixing between the neutral scalars described by the real parts of φ_1^0 and φ_2^0 . It provides in particular, in the large $\tan\beta$ limit for which v_1 is small, a rather large mass² term for h_1 contributing to $m^2(H^\pm \simeq h_1^\pm)$, m_A^2 and to a small mixing between the neutral scalars.

Specializing in the MSSM, adding the supersymmetric (m_W^2, m_Z^2) and supersymmetry-breaking contributions to the mass² matrices implies immediately, in this specific model,

$$m_{H^\pm}^2 = m_W^2 + m_A^2. \quad (93)$$

In the large $\tan\beta$ limit, $h \simeq z \simeq \sqrt{2} \Re \varphi_2^0$ and $H \simeq \sqrt{2} \Re \varphi_1^0$ have masses close to m_Z and m_A , respectively.

The mass² matrix for the neutral scalar fields h_1^0 and h_2^0 may be written as the sum of two supersymmetry-conserving and supersymmetry-breaking contributions. It follows from (79) involving $z = \sqrt{2} \Re (-h_1^0 \cos\beta +$

$h_2^0 \sin\beta)$, and for the non-supersymmetric part (90) involving $\sqrt{2} \Re (h_1^0 \sin\beta - h_2^0 \cos\beta)$:

$$\mathcal{M}_o^2 = \underbrace{\begin{pmatrix} m_Z^2 & 0 \\ 0 & 0 \end{pmatrix}}_{-\beta} + \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & m_A^2 \end{pmatrix}}_{\beta}. \quad (94)$$

SUSY-conserving SUSY-breaking

The two basis denoted by $_{-\beta}$ and $_\beta$ are rotated from the (h_1^0, h_2^0) basis by angles $-\beta$ and β , and are at angle 2β . The mass of the lightest eigenstate increases from 0 to m_Z when β increases from $\pi/4$ to $\pi/2$ (or decreases from $\pi/4$ to 0), assuming $m_A > m_Z$. The second derivative of V , in the SM-like direction orthogonal to the m_A^2 eigenstate of the second matrix, at angle 2β with the direction of z , receives only a contribution $m_Z^2 \cos^2 2\beta$ from the first term. This implies a mass eigenstate verifying

$$m_h \leq m_Z |\cos 2\beta| \quad (+ \text{radiative corr.}). \quad (95)$$

More precisely (94) reads

$$\mathcal{M}_o^2 = \begin{pmatrix} c_\beta^2 m_Z^2 + s_\beta^2 m_A^2 & -s_\beta c_\beta (m_Z^2 + m_A^2) \\ -s_\beta c_\beta (m_Z^2 + m_A^2) & s_\beta^2 m_Z^2 + c_\beta^2 m_A^2 \end{pmatrix}, \quad (96)$$

and has the eigenvalues

$$m_{H,h}^2 = \frac{m_Z^2 + m_A^2}{2} \pm \sqrt{\left(\frac{m_Z^2 + m_A^2}{2}\right)^2 - m_Z^2 m_A^2 \cos^2 2\beta}. \quad (97)$$

The smallest one verifies (95), approaching m_Z in the large $\tan\beta$ limit for which the two mass eigenvalues get close to m_Z and m_A as seen from (94). The lighter scalar h becomes close to being the spin-0 partner of the Z , $h \simeq z \simeq \sqrt{2} \Re h_2^0$, with a mass close to m_Z , the heavier one $H \simeq \sqrt{2} \Re h_1^0$ having a mass close to m_A .

These formulas, leading back to $m_{H^\pm} = m_W$ and $m_H = m_Z$ when the supersymmetry-breaking parameter m_A^2 vanishes, in agreement with (82,86), illustrate the implications of gauge/BE-Higgs unification, even in a situation of broken supersymmetry. Large radiative corrections, involving most notably very heavy and/or strongly mixed stop quarks, are then required in the MSSM to keep a chance to get m_h sufficiently above m_Z , in view of identifying it with the 125 GeV spin-0 boson.

B. Spin-0 masses in the N/nMSSM

with heavier spin-0 bosons thanks to the extra singlet

The situation is much better in the N/nMSSM (or also in the USSM) thanks to the trilinear coupling λ in the superpotential leading to an additional quartic term in the potential $\lambda^2 |h_2 h_1|^2$, and to a steepest potential allowing for larger masses, already at the classical level.

Indeed starting from the R -symmetric nMSSM spectrum (73) with $m_A = \lambda v/\sqrt{2}$, $m_{H^\pm} = m_W$ and $m_h = m_Z$, independently of β [1], the sum of supersymmetric and supersymmetry-breaking contributions leads to the mass formulas

$$\begin{cases} m_A^2 = \frac{\lambda^2 v^2}{2} + \delta m_A^2, \\ m_{H^\pm}^2 = m_W^2 + \delta m_A^2 = m_W^2 + m_A^2 - \frac{\lambda^2 v^2}{2}. \end{cases} \quad (98)$$

Neutral scalars are also expected to be heavier than in the MSSM. Their 2×2 mass² submatrix, restricted to the h_1^0, h_2^0 subspace by ignoring the singlet scalar, now reads

$$\mathcal{M}_o^2 = \underbrace{\begin{pmatrix} m_Z^2 & 0 \\ 0 & \frac{\lambda^2 v^2}{2} \end{pmatrix}}_{\text{SUSY-conserving (nMSSM)}}_{-\beta} + \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & \delta m_A^2 \end{pmatrix}}_{\text{SUSY-breaking}}_\beta. \quad (99)$$

Its lightest mass eigenstate is immediately seen to be in the range $(m_Z, \lambda v/\sqrt{2})$. For

$$\lambda \geq \sqrt{\frac{g^2 + g'^2}{2}} = \frac{m_Z \sqrt{2}}{v} = 2^{3/4} G_F^{1/2} m_Z \simeq .52 \quad (100)$$

the two mass eigenstates are already both heavier than m_Z , independently of $\tan\beta$ and before taking into account supersymmetry-breaking effects from δm_A^2 . This makes it much easier to reach 125 GeV, without having to rely on large radiative corrections from very heavy stop quarks, as one must do in the MSSM.

For $\beta = \pi/4$ i.e. $v_1 = v_2$ the 2×2 matrix (99) has $\lambda^2 v^2/2$ and $m_Z^2 + \delta m_A^2$ for eigenvalues, its lightest mass eigenvalue being large if $\lambda^2 v^2/2$ and δm_A^2 are both large. We then get

$$\begin{cases} m_z^2 = m_Z^2 + \delta m_A^2, \\ m_{H^\pm}^2 = m_W^2 + \delta m_A^2 \end{cases} \quad (101)$$

as obtained for example with gravity-induced supersymmetry-breaking, for $\delta m_A^2 = 4 m_{3/2}^2$ [62]. This illustrates how the gauge/BE-Higgs unification may manifest on the mass spectrum. For example taking here m_z at 125 GeV would imply a H^\pm around 117 GeV. In the absence of a relatively light H^\pm , the above relations would lead us to view, for $\tan\beta = 1$, the z as a heavier spin-0 BEH boson close in mass to the H^\pm , rather than the one found at 125 GeV.

More precisely the second derivative of V , in the SM-like direction orthogonal to the m_A^2 eigenstate of the second matrix, at angle 2β with the direction of z , receives a contribution only from the first matrix in (99), and is thus now equal to $m_Z^2 \cos^2 2\beta + \frac{\lambda^2 v^2}{2} \sin^2 2\beta$. This implies a neutral mass eigenstate verifying

$$m_h^2 \leq m_Z^2 \cos^2 2\beta + \frac{\lambda^2 v^2}{2} \sin^2 2\beta \quad (+ \text{radiative corr.}), \quad (102)$$

This upper bound may also be obtained directly by noting that the neutral SM-like combination $\varphi_{\text{sm}}^\circ = h_1^\circ \cos\beta + h_2^\circ \sin\beta$ has its quartic coupling $\lambda_{\text{sm}} |\varphi_{\text{sm}}^\circ|^4$ in the N/nMSSM potential (60) or (62) fixed by

$$\lambda_{\text{sm}} = \frac{g^2 + g'^2}{8} \cos^2 2\beta + \frac{\lambda^2}{4} \sin^2 2\beta. \quad (103)$$

Expanding V as a function of $\sqrt{2} \Re \varphi_{\text{sm}}^\circ$ provides for this field the mass² parameter $2\mu_{\text{sm}}^2 = 2\lambda_{\text{sm}} v^2 = m_Z^2 \cos^2 2\beta + \frac{\lambda^2 v^2}{2} \sin^2 2\beta$. Neutral spin-0 bosons cannot all be heavier, the lightest having mass $\leq \mu_{\text{sm}} \sqrt{2}$ at most, leading to the mass bound (102).

Let us explicitate for completeness the mass² matrix (99) for the neutral scalar h_1^0, h_2^0 subspace:

$$\begin{pmatrix} c_\beta^2 m_Z^2 + s_\beta^2 (\frac{\lambda^2 v^2}{2} + \delta m_A^2) & -s_\beta c_\beta (m_Z^2 - \frac{\lambda^2 v^2}{2} + \delta m_A^2) \\ -s_\beta c_\beta (m_Z^2 - \frac{\lambda^2 v^2}{2} + \delta m_A^2) & s_\beta^2 m_Z^2 + c_\beta^2 (\frac{\lambda^2 v^2}{2} + \delta m_A^2) \end{pmatrix}, \quad (104)$$

with $m_A^2 = \frac{\lambda^2 v^2}{2} + \delta m_A^2$ as in (98). m_Z^2 and $\frac{\lambda^2 v^2}{2}$ correspond to the supersymmetric contributions in the R -symmetric nMSSM spectrum (73) [1], δm_A^2 being the supersymmetry-breaking contribution from the “inert doublet” φ_{in} mass term, as in (90). This 2×2 submatrix should then be included within a 3×3 matrix taking into account mixing effects with the singlet $\sqrt{2} \Re s$, involving in particular $\mu \lambda v_1/\sqrt{2}$ and $\mu \lambda v_2/\sqrt{2}$ as seen from (62).

A more instructive expression of the matrix (99,104) is obtained in the SUSY basis $-\beta$, by rotating by -2β the matrix for the supersymmetry-breaking contribution; or by writing $-\Re \varphi_{\text{in}} = \Re (-h_1^0 \sin\beta + h_2^0 \cos\beta) = \sin 2\beta \Re (-h_1^0 \cos\beta + h_2^0 \sin\beta) + \cos 2\beta \Re (h_1^0 \sin\beta + h_2^0 \cos\beta)$:

$$\mathcal{M}_o^2 = \begin{pmatrix} m_Z^2 + \delta m_A^2 \sin^2 2\beta & \delta m_A^2 \sin 2\beta \cos 2\beta \\ \delta m_A^2 \sin 2\beta \cos 2\beta & \frac{\lambda^2 v^2}{2} + \delta m_A^2 \cos^2 2\beta \end{pmatrix}_{-\beta}. \quad (105)$$

C. Charginos and neutralinos mass matrices

*as understood from R symmetry
and gauge/BE-Higgs unification*

The higgsino fields \tilde{h}_{1L} and \tilde{h}_{2L} , described by H_1 and H_2 with $R = 0$, transform according to (37) as $\tilde{h}_{iL} \rightarrow e^{-i\alpha} \tilde{h}_{iL}$, $\tilde{h}_{2L} \rightarrow e^{-i\alpha} \tilde{h}_{2L}$. The Dirac higgsino doublet ψ constructed from \tilde{h}_{1L} and $(\tilde{h}_{2L})^c$ and the Majorana gauginos λ transform chirally in opposite ways as in (41,42). Gaugino mass terms $m_{1/2}$ (denoted by m_3 , m_2 and m_1 for the gluinos and $SU(2) \times U(1)$ gauginos) violate the continuous R symmetry, as for a μ term, reducing it to R -parity [32, 53].

Gaugino and higgsino fields can combine through R -invariant non-diagonal mass terms generated from the

Yukawa couplings (39). The resulting charginos and neutralinos appear at this stage, with $\mu = m_i = 0$, as *Dirac particles* carrying $R = \pm 1$ as expressed in (42), leading to the $U(1)_R$ symmetric nMSSM mass spectrum (72,73).

The Dirac zino with $R = +1$ of mass m_Z , in agreement with the as-yet-unbroken supersymmetry still present in the neutral sector, is obtained by combining

$$\begin{cases} \text{the gaugino} & \lambda_Z = \lambda_3 c_\theta - \lambda' s_\theta, \\ \text{the higgsino} & -\tilde{h}_z = \tilde{h}_1^0 c_\beta - \tilde{h}_2^0 s_\beta. \end{cases} \quad (106)$$

The gaugino λ_Z is directly associated with the Z , and the higgsino \tilde{h}_z is described by the chiral superfield H_z in (77). The Dirac zino may be expressed as in [1] as

$$\lambda_{ZL} + (-\tilde{h}_z)_R = (\lambda_3 c_\theta - \lambda' s_\theta)_L + (\tilde{h}_1^0 c_\beta - \tilde{h}_2^0 s_\beta)_R, \quad (107)$$

or reexpressed in terms of the two Majorana spinors in (106). The corresponding 2×2 mass matrix in a gaugino-higgsino basis,

$$\mathcal{M}_{\text{zinos}} = \begin{pmatrix} 0 & m_Z \\ m_Z & 0 \end{pmatrix}, \quad (108)$$

may be further unpacked into a 4×4 matrix as below in (110).

Including the $\Delta R = \pm 2$ gaugino and higgsino mass terms m_i and μ breaking explicitly R symmetry, we get, with $\mu H_2 H_1 = \mu (H_2^+ H_1^- - H_2^0 H_1^0)$, the chargino and neutralino mass matrices in the MSSM,

$$\mathcal{M}_{\text{winos}} = \begin{pmatrix} m_2 & m_W \sqrt{2} s_\beta \\ m_W \sqrt{2} c_\beta & \mu \end{pmatrix}, \quad (109)$$

and

$$\mathcal{M}_{\text{inos}} = \begin{pmatrix} m_1 & 0 & -s_\theta c_\beta m_Z & s_\theta s_\beta m_Z \\ 0 & m_2 & c_\theta c_\beta m_Z & -c_\theta s_\beta m_Z \\ -s_\theta c_\beta m_Z & c_\theta c_\beta m_Z & 0 & -\mu \\ s_\theta s_\beta m_Z & -c_\theta s_\beta m_Z & -\mu & 0 \end{pmatrix}. \quad (110)$$

The part proportional to m_Z in the neutralino mass matrix is the supersymmetric contribution, in a way compatible with gauge/BE-Higgs unification, while the $\Delta R = \pm 2$ part involving m_1 , m_2 and μ is the supersymmetry-breaking part. We recall that the parameter μ , although initially “supersymmetric”, still leads to supersymmetry-breaking effects in the presence of v_1 and v_2 .

Their eigenvalues verify relations such as

$$m^2(wino_1) + m^2(wino_2) = 2 m_W^2 + \mu^2 + m_2^2, \quad (111)$$

and, similarly,

$$\sum_{1..4} m^2(\text{neutralino}) = 2 m_Z^2 + 2 \mu^2 + m_1^2 + m_2^2. \quad (112)$$

Without the gaugino masses m_1 , m_2 , and with the dimension-2 soft-breaking mass terms for h_1 and h_2 generated spontaneously as in (63) so that $m_A^2 = 2 \mu^2$, the average mass² for bosons and fermions would be the same in the multiplets considered, with

$$\left\{ \begin{array}{l} 3 m_W^2 + m_{H^\pm}^2 = 4 m_W^2 + m_A^2 = 4 m_W^2 + 2 \mu^2 \\ \qquad \qquad \qquad = 2 [m^2(wino_1) + m^2(wino_2)] \\ 3 m_Z^2 + m_h^2 + m_H^2 + m_A^2 = 4 m_Z^2 + 2 m_A^2 = 4 m_Z^2 + 4 \mu^2 \\ \qquad \qquad \qquad = 2 \sum_{1..4} m^2(\text{neutralino}). \end{array} \right. \quad (113)$$

D. Neutralinos in the N/nMSSM

as understood from R symmetry
and gauge/BE-Higgs unification

The N/nMSSM introduces an additional neutral singlino described by S . The chargino mass matrix (109) is simply affected by the replacement $\mu \rightarrow \mu_{\text{eff}} = \mu + \lambda <s>$. The neutralino mass matrix (110) gets embedded into a 5×5 one. It now includes R -conserving non-diagonal mass terms corresponding to the nMSSM mass spectrum in (72), with $-\frac{\lambda v}{\sqrt{2}} \sin \beta$ and $-\frac{\lambda v}{\sqrt{2}} \cos \beta$ contributions obtained from the R -invariant $\lambda H_2 H_1 S$ coupling (75) mixing the doublet higgsinos \tilde{h}_1 and \tilde{h}_2 with the singlino ζ . \tilde{h}_{1R} , \tilde{h}_{2R} and the singlino ζ_L all have $R = 1$, in agreement with the R transformation properties

$$\text{gauginos: } \lambda \xrightarrow{R} e^{\gamma_5 \alpha} \lambda; \quad \tilde{h}_i \xrightarrow{R} e^{-\gamma_5 \alpha} \tilde{h}_i \quad \zeta \xrightarrow{R} e^{\gamma_5 \alpha} \zeta. \quad (114)$$

The R -conserving part of the mass matrix corresponds to a conserved supersymmetry. It is a rank-4 5×5 matrix obtained by unpacking the two matrices

$$\begin{pmatrix} 0 & m_Z \\ m_Z & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & -\frac{\lambda v}{\sqrt{2}} \\ -\frac{\lambda v}{\sqrt{2}} & 0 \end{pmatrix}. \quad (115)$$

This provides as in (72) 5 neutralinos organized as a Dirac zino of mass m_Z , a Dirac neutralino mixing the singlino with the left-over higgsino, carrying $R = \pm 1$, and a massless chiral photino with $R = 1$. The massive spinors involve, as in (107) [1]

$$\left\{ \begin{array}{ll} \lambda_Z = \lambda_3 c_\theta - \lambda' s_\theta & \text{combined with } -\tilde{h}_z = \tilde{h}_1^0 c_\beta - \tilde{h}_2^0 s_\beta, \\ \tilde{s} & \text{combined with } -(\tilde{h}_1^0 s_\beta + \tilde{h}_2^0 c_\beta). \end{array} \right. \quad (116)$$

The neutralino mass spectrum (72) for the $U(1)_R$ -invariant nMSSM [1] is reexpressed into the 5×5 neutralino

mass matrix

$$\begin{pmatrix} 0 & 0 & -s_\theta c_\beta m_Z & s_\theta s_\beta m_Z & 0 \\ 0 & 0 & c_\theta c_\beta m_Z & -c_\theta s_\beta m_Z & 0 \\ -s_\theta c_\beta m_Z & c_\theta c_\beta m_Z & 0 & 0 & -\frac{\lambda v}{\sqrt{2}} s_\beta \\ s_\theta s_\beta m_Z & -c_\theta s_\beta m_Z & 0 & 0 & -\frac{\lambda v}{\sqrt{2}} c_\beta \\ 0 & 0 & -\frac{\lambda v}{\sqrt{2}} s_\beta & -\frac{\lambda v}{\sqrt{2}} c_\beta & 0 \end{pmatrix}. \quad (117)$$

One completes this R -symmetric 5×5 mass matrix by the $\Delta R = \pm 2$ contributions, re-introducing the μ and μ_S doublet and singlet mass parameters previously discarded from the nMSSM superpotential [1] to get R symmetry. If a translation on S has to be performed, μ and μ_S get modified into $\mu_{\text{eff}} = \mu + \lambda <s>$ and $\mu_{S\text{ eff}} = \mu_S + 2\kappa <s>$. The gaugino mass parameters m_1 and m_2 may be generated by radiative corrections, or from gravity-induced supersymmetry breaking as e.g. in [62].

The resulting 5×5 neutralino mass matrix reads

$$\begin{pmatrix} m_1 & 0 & -s_\theta c_\beta m_Z & s_\theta s_\beta m_Z & 0 \\ 0 & m_2 & c_\theta c_\beta m_Z & -c_\theta s_\beta m_Z & 0 \\ -s_\theta c_\beta m_Z & c_\theta c_\beta m_Z & 0 & -\mu_{\text{eff}} & -\frac{\lambda v}{\sqrt{2}} s_\beta \\ s_\theta s_\beta m_Z & -c_\theta s_\beta m_Z & -\mu_{\text{eff}} & 0 & -\frac{\lambda v}{\sqrt{2}} c_\beta \\ 0 & 0 & -\frac{\lambda v}{\sqrt{2}} s_\beta & -\frac{\lambda v}{\sqrt{2}} c_\beta & \mu_{S\text{ eff}} \end{pmatrix}. \quad (118)$$

It involves 12 non-diagonal terms $\propto m_Z$ and $\lambda v/\sqrt{2}$, originating from the supersymmetric and $U(1)_R$ -invariant nMSSM mass spectrum (72,117), next to the 5 additional $\Delta R = \pm 2$ gaugino and doublet + singlet higgsino mass terms. The chargino and neutralino masses now verify

$$m^2(\text{wino}_1) + m^2(\text{wino}_2) = 2m_W^2 + \mu_{\text{eff}}^2 + m_2^2, \quad (119)$$

and

$$\sum_{1...5} m^2(\text{ino}) = 2 \left(m_Z^2 + \frac{\lambda^2 v^2}{2} + \mu_{\text{eff}}^2 \right) + m_1^2 + m_2^2 + \mu_{S\text{ eff}}^2, \quad (120)$$

reducing to (111,112) for $\lambda = 0$.

E. Squarks, sleptons and supersymmetry breaking

Left-handed quark and lepton fields are described by the left-handed chiral quark and lepton doublet superfields, Q and L . Right-handed ones, viewed as the conjugates of antiquark and antilepton fields, are described by the left-handed singlet superfields \bar{U} , \bar{D} and \bar{E} . H_1 and H_2 generate charged-lepton and down-quark masses, and

up-quark masses, from the trilinear superpotential couplings \mathcal{W}_{lq} of lepton and quark superfields to H_1 and H_2 [2],

$$\mathcal{W}_{lq} = \lambda_e H_1 \bar{E} L + \lambda_d H_1 \bar{D} Q - \lambda_u H_2 \bar{U} Q. \quad (121)$$

H_1 and H_2 are separately responsible, from $< h_1^0 > = v_1/\sqrt{2}$, $< h_2^0 > = v_2/\sqrt{2}$, for charged-lepton and down-quark masses, and up-quark masses, respectively, with

$$m_e = \frac{\lambda_e v_1}{\sqrt{2}}, \quad m_d = \frac{\lambda_d v_1}{\sqrt{2}}, \quad m_u = \frac{\lambda_u v_2}{\sqrt{2}}. \quad (122)$$

This tends to favor a smaller v_1 as compared to v_2 i.e. a large $\tan\beta = v_2/v_1$, in view of the large mass of the t quark as compared to the b .

The superpotential interactions resulting from (121) are invariant under the continuous $U(1)_R$ symmetry (36,37), with $R = +1$ for left-handed (anti)quark and (anti)lepton superfields, so that leptons and quarks carry $R = 0$ and sleptons ans squarks $+1$ (for \tilde{l}_L, \tilde{q}_L) or -1 (for \tilde{l}_R, \tilde{q}_R). Gauge and chiral superfields transform under R according to (71) so that, altogether

$$\begin{cases} V_a & \xrightarrow{R} V_a(x, \theta e^{-i\alpha}, \bar{\theta} e^{i\alpha}), \\ (L, Q; \bar{E}, \bar{D}, \bar{U}) & \xrightarrow{R} e^{i\alpha} (L, Q; \bar{E}, \bar{D}, \bar{U})(x, \theta e^{-i\alpha}), \\ H_{1,2} & \xrightarrow{R} H_{1,2}(x, \theta e^{-i\alpha}), \\ S & \xrightarrow{R} e^{2i\alpha} S(x, \theta e^{-i\alpha}), \end{cases} \quad (123)$$

the superpotential \mathcal{W} , including \mathcal{W}_{lq} , having $R = 2$.

A mixing between left-handed doublet and right-handed singlet squarks, e.g. \tilde{t}_L and \tilde{t}_R , with $R = +1$ and -1 respectively, can be generated by the $\Delta R = \pm 2$ terms in the Lagrangian density. It leads for the \tilde{t} squarks to a non-diagonal term $A_t - \mu_m t \cot\beta$, combining the contributions from the dimension-3 soft-breaking terms involving h_2 with those from F terms.

One essential question is the way by which sleptons and sleptons can acquire very large masses. A spontaneous breaking of the global supersymmetry generates a massless chiral Goldstone spinor, carrying $R = 1$ as seen in (70). It may describe the photino, or more generally a neutral gaugino possibly involving an extra- $U(1)$ gaugino, combined with a neutral spin- $\frac{1}{2}$ field ζ described by a $R = 2$ chiral superfield [2].

An extension of the gauge group to include an extra $U(1)$ factor is necessary if one intends to generate large masses for all sleptons and sleptons, at the tree approximation, in a globally supersymmetric theory. Indeed with $SU(3) \times SU(2) \times U(1)$ or $SU(5)$ as the gauge group the sleptons of the first generation would verify, at the classical level [4],

$$\sum_{1,2} m^2(\tilde{u}_i) + m^2(\tilde{d}_i) = 2(m_u^2 + m_d^2). \quad (124)$$

One at least should then have a very small or even negative mass², leading to a charge and color-breaking vacuum. This leads either to consider an extension of the gauge group to include an extra- $U(1)$ factor, or to generate large masses from radiative corrections as in the “gauge-mediated” supersymmetry-breaking (GMSB) models, or to go to local supersymmetry. Or, finally, to move to extra dimensions to generate the required breaking of the supersymmetry using *discrete boundary conditions involving R-parity*, as discussed in Section IX.

Making all squarks and sleptons heavy was first done through the v.e.v. of the D component from an extra $U(1)$ with non-vanishing axial couplings to quarks and leptons, as in the USSM briefly introduced in subsection IV B. This may be done by taking advantage of the U transformations [32,33,53,56], under which

$$\left\{ \begin{array}{ccc} (L, Q; \bar{E}, \bar{D}, \bar{U}) & \xrightarrow{U} & e^{-\frac{i\alpha}{2}} (L, Q; \bar{E}, \bar{D}, \bar{U}), \\ H_{1,2} & \xrightarrow{U} & e^{i\alpha} H_{1,2}, \\ S & \xrightarrow{U} & e^{-2i\alpha} S. \end{array} \right. \quad (125)$$

This is in particular a symmetry of the trilinear superpotential \mathcal{W}_{lq} responsible for quarks and lepton masses [2]. It acts axially on quarks and leptons, the corresponding (axionlike) Goldstone boson (a) getting eliminated as the extra neutral boson Z' (also called U) acquires a mass. This still leaves us with the question of how to generate a mass for gluinos, possibly from radiative corrections [53], or from supergravity as anticipated in [32], since supergravity requires abandoning the continuous R symmetry in favor of the discrete R -parity.

Supersymmetry breaking is now usually realized by generating soft supersymmetry-breaking terms [27, 58] from radiative corrections, or supergravity. In the first case, gauge-mediated models are generally characterized by the possibility of a light or very light gravitino LSP, behaving very much like a goldstino [32].

When the local supersymmetry is spontaneously broken, it generates a massive spin- $\frac{3}{2}$ Majorana gravitino. Its mass term $m_{3/2}$ breaks explicitly the continuous R symmetry ($U(1)_R$), reducing it to R -parity. This re-allows, in the supergravity framework, direct gaugino mass parameters, whose mass scale may be naturally fixed from the gravitino mass $m_{3/2}$. Soft supersymmetry-breaking terms may then be generated from supergravity [59], leading to gravity-induced supersymmetry-breaking models [60–64], where the gravitino is generally taken to be heavy.

In all these cases (MSSM, N/nMSSM, USSM, ...) supersymmetry-breaking and R -symmetry breaking contributions are added to the Lagrangian density. This includes the reintroduction of the “supersymmetric” parameter μ (with $\Delta R = \pm 2$ and also breaking the extra- $U(1)$ symmetry), possibly regenerated from a translation of S , and the inclusion of the $\Delta R = \pm 2$ gaugino mass pa-

rameters, and of other terms of dimension ≤ 3 breaking supersymmetry explicitly but softly.

This would be a natural place to stop this presentation of the Supersymmetric Standard Model, hoping for supersymmetric particles to be discovered soon at LHC.

Still there may be more, and this is likely not to be the end of the story. More symmetries may be jointly operating to provide a better understanding of the electroweak and grand-unification breakings, opening the way to new compact dimensions of space-time, next to the quantum anticommuting dimensions of supersymmetry.

VII. BEYOND THE N/nMSSM

A. Towards a $N = 2$ supersymmetric spectrum

Let us return to the R -symmetric superpotential (50), $\mathcal{W}_{nMSSM} = S(\lambda H_2 H_1 + \sigma)$ [1] and resulting spectrum (72,73,115,117). For λ equal to the limiting value (100),

$$\lambda = \sqrt{\frac{g^2 + g'^2}{2}} \simeq .52 \quad (126)$$

(up to a possible convention-dependent sign for a real coupling), so that

$$m_Z = \frac{\sqrt{g^2 + g'^2} v}{2} = \frac{\lambda v}{\sqrt{2}}, \quad (127)$$

the theory has in its Z sector, with the superpotential

$$\mathcal{W}_o = - \sqrt{\frac{g^2 + g'^2}{2}} H_2^0 H_1^0 S + \sigma S, \quad (128)$$

an unbroken $N = 2$ supersymmetry, independently of the value of the mixing angle β [43, 66]. Its effects may be observed in the two 2×2 neutralino mass matrices (115), which participate equally in the 5×5 neutralino mass matrix (117). The Z gets associated with 2 Dirac zinos (or 4 Majorana ones) and 5 neutral spin-0 bosons within a massive gauge multiplet of $N = 2$ supersymmetry, according to

$$nMSSM \text{ with } \lambda = \sqrt{\frac{g^2 + g'^2}{2}} \Rightarrow$$

$$Z \stackrel{SUSY}{\iff} 4 \text{ Majorana zinos} \stackrel{SUSY}{\iff} 5 \text{ spin-0 bosons.}$$

This is valid as long as supersymmetry remains unbroken in this sector. This massive $U(1)_R$ symmetric nMSSM spectrum even presents, for the W^\pm and Z multiplets, an effective $N = 2$ supersymmetry, with in this case the electric charge acting as a central charge [44], this $N = 2$ supersymmetry being broken in the W^\pm multiplet for $\tan \beta \neq 1$.

The $N = 2$ supersymmetry may be extended to gluons and gluinos, if the latter are turned into Dirac particles, and accompanied by a complex octet of spin-0 “sgluons” [53]. If we want to pursue in this direction of $N = 2$ supersymmetry we must introduce, next to the singlet S , adjoint $SU(2)$ and $SU(3)$ chiral superfields T and O with trilinear super-Yukawa couplings fixed by the gauge couplings,

$$\lambda_i = g_i \sqrt{2}, \quad \lambda' = \frac{g'}{\sqrt{2}}. \quad (130)$$

The electroweak couplings of H_1 and H_2 to the singlet S and triplet T are given by the superpotential (22) already encountered for the F -breaking of the supersymmetry [30, 43],

$$\mathcal{W} = \frac{1}{\sqrt{2}} H_2 (g \tau \cdot T - g' S) H_1 + \sigma S. \quad (131)$$

This $N = 2$ superpotential includes in particular, precisely, the nMSSM-type one

$$\mathcal{W}_o = - \sqrt{\frac{g^2 + g'^2}{2}} H_2^0 H_1^0 S_Z + \sigma S \quad (132)$$

for the chiral superfield $S_Z = \cos \theta T_3 - \sin \theta S$ associated with the Z , as written earlier in (128). This provides an $N = 2$ interpretation for the mass degeneracy occurring at the classical level, for unbroken supersymmetry, between the 5 neutral spin-0 bosons of the nMSSM, all of mass m_Z for $\lambda = \sqrt{(g^2 + g'^2)/2}$ [66].

One must also consider 4 doublet chiral superfields (or $SU(5)$ quintuplets) instead of 2, by introducing H'_1 and H'_2 next to H_1 and H_2 to provide the required degrees of freedom for constructing 4 Dirac winos, so that

$$W^\pm \stackrel{SUSY}{\iff} 4 \text{ Dirac winos} \stackrel{SUSY}{\iff} 5 \text{ charged spin-0 bosons}. \quad (133)$$

The W^\pm and Z are then associated with 5 charged and 5 neutral spin-0 bosons, all of masses m_W and m_Z as long as the $N = 2$ supersymmetry remains unbroken. This ultimately provides the spectrum for the gauge-and-BE-Higgs sector of $N = 2$ supersymmetric grand-unified theories [45], which may then be formulated in a 5 or 6-dimensional space-time [46].

B. Radiative gluino masses from messenger quarks

Gluinos being Majorana particles transforming chirally as in (37), a continuous R symmetry would forbid gluino masses, except if gluinos are turned into Dirac particles, as would be the case, precisely, within a $N = 2$ theory as we discussed.

But let us return to $N = 1$ for a moment. Gluinos are massless at lowest order, within global supersymmetry. Still if one abandons the continuous R symmetry

one may consider generating *radiatively* gluino masses from their couplings to a new set of massive *messenger quarks* described by the chiral superfields \mathcal{Q} and $\bar{\mathcal{Q}}$, *vectorially coupled* to standard model particles and sensitive to the source of supersymmetry-breaking, for messenger squarks and quarks to have different masses [53]. One also needs to introduce a source of breaking for the continuous R -symmetry, otherwise gluinos would still remain massless.

This requires some care especially if we intend to appeal to the F -mechanism for breaking spontaneously supersymmetry [30, 31], as the presence of an R symmetry is needed for a generic breaking of the supersymmetry as discussed in subsection III D. At the same time however, R symmetry must be broken to get gluino masses, then leading to a massless *R-Goldstone-boson* or light *R-axion*. This may lead to prefer generating the spontaneous breaking of the supersymmetry in the messenger sector through the extra- $U(1)$ gauge interactions of the messenger (s)quarks with a non-vanishing $\langle D \rangle$, as done in [53].

Let us consider a second octet of paragluinos ζ_a , described by a chiral octet superfield O with $R = 0$. One may then generate radiatively a gluino mass in a R -symmetric way, with the two Majorana octets transforming in opposite ways according to

$$\begin{cases} \text{gluinos } \lambda_a & \xrightarrow{R} e^{\gamma_5 \alpha} \lambda_a, \\ \text{paragluinos } \zeta_a & \xrightarrow{R} e^{-\gamma_5 \alpha} \zeta_a. \end{cases} \quad (134)$$

These two octets could originate, together with their associated spin-0 gluons, now often called *sgluons*, from an underlying $N = 2$ supersymmetry [43, 44]. They are described by a chiral octet superfield O with $R = 0$ (rather than 2, so as to lead to (134)), coupled to *massive messenger quark superfields* \mathcal{Q} and $\bar{\mathcal{Q}}$ with $R = 1$, themselves vectorially coupled to standard model particles.

These messenger quark superfields interact with the octet O through the $R = 2$ superpotential [53]

$$\mathcal{W}_{\text{mess.}} = m_{\mathcal{Q}} \bar{\mathcal{Q}} \mathcal{Q} + \lambda_O \bar{\mathcal{Q}} O \mathcal{Q}, \quad (135)$$

color indices being omitted for simplicity. This generates a R -conserving Dirac mass term (m_D) for the Dirac gluino octet with $R = 1$,

$$\text{Dirac gluinos} = \lambda_{aL} + \zeta_{aR}, \quad (136)$$

through one-loop diagrams involving the massive messenger quarks and squarks, sensitive to the source of spontaneous supersymmetry breaking, e.g. through an extra $U(1)$.

We still have to pay attention to the two additional real octets of spin-0 gluons described by O (sgluons), as one tends to acquire a negative mass² from quantum corrections. This instability may be avoided by introducing

some amount of R -symmetry breaking, to locally stabilize the vacuum through a superpotential Majorana mass term $\frac{1}{2}\mu_O O^2$ for the second gluino octet ζ . The resulting explicit breaking of R symmetry may be, again, a reason to prefer breaking spontaneously supersymmetry in the messenger sector through D terms, rather than through the F -breaking mechanism making use of R symmetry.

This leads for gluinos to both Dirac and Majorana mass terms, with a *see-saw* type 2×2 mass matrix [53]

$$\mathcal{M}_{\text{gluinos}} = \begin{pmatrix} 0 & m_D \\ m_D & \mu_O \end{pmatrix}, \quad (137)$$

a mechanism introduced for gluinos even before it started getting widely considered for neutrinos. To be general a direct Majorana mass term m_3 for ordinary gluinos could still be added, although the purpose of this study was to discuss how an effective m_3 could be generated radiatively from the above see-saw type mass matrix.

This still leaves us, however, with a vacuum state that is locally-stable but only *metastable*, with a lower energy vacuum state for which color would be spontaneously broken [53]. Fortunately this metastable vacuum is in practice effectively stable. The possible interest of such metastable vacuum states, that had escaped attention at the time, was brought back to consideration more recently [67]. To generate radiatively in this way a significant mass for the gluinos, which must now be \sim TeV scale at least as the result of LHC experiments [68, 69], it is necessary to consider quite high values for the messenger quark masses.

One may also imagine gauging the R symmetry, eliminating the corresponding Goldstone boson if R is spontaneously broken. This would lead to *a new force acting only on supersymmetric particles* (still to be discovered), and therefore, presumably, on dark matter. This new force may even be long-ranged if R symmetry stayed unbroken, otherwise it would have a finite range \hbar/mc where m is the mass of the corresponding gauge boson associated with R symmetry.

VIII. $N = 2$ SUPERSYMMETRIC GRAND-UNIFIED THEORIES

A. Moving to higher dimensions

We saw that a $N = 2$ supersymmetric theory with a massless matter hypermultiplet in the adjoint representation provides the $N = 4$ supersymmetric Yang-Mills theory, the adjoint gauge superfield interacting with 3 adjoint chiral ones coupled through the trilinear superpotential (25) $\mathcal{W} = g\sqrt{2}f_{ijk}S_1^iS_2^jS_3^k$, describing 1 spin-1 + 4 spin- $\frac{1}{2}$ + 6 spin-0 adjoint fields, with a $SU(4)_R \sim O(6)$ group acting on the 4 supersymmetry generators [43, 44].

These theories, also obtained from the low-energy region of the dual spinor model [70], or from the dimensional reduction of a supersymmetric Yang-Mills theory in 10 dimensions [71], are remarkably elegant but very constrained, and more difficult to apply to fundamental particles and interactions. Still the extra dimensions of space-time may well be at the origin of the breaking of both the supersymmetry and the grand-unification symmetry, as we shall see.

Starting again from $N = 1$ supersymmetry in 4 dimensions, with gaugino and higgsino mass terms related by $m_1 = m_2 = m_3 = -\mu$, possibly also equal to the gravitino mass $m_{3/2}$, and assuming $\tan\beta = 1$ for simplicity, we get, directly or from (109,110), remarkable mass relations like [72]

$$\begin{cases} m^2(\text{winos}) &= m_W^2 + m_{3/2}^2, \\ m^2(\text{zinos}) &= m_Z^2 + m_{3/2}^2, \\ m(\text{photino}) &= m(\text{gluinos}) = m_{3/2}, \end{cases} \quad (138)$$

up to radiative corrections.

These formulas, obtained in 4 dimensions, already point to a higher-dimensional origin of the $m_{3/2}^2$ contributions to the 4d mass², with supersymmetric particles carrying momenta $\pm m_{3/2}$ along an extra compact dimension. This leads us to consider again theories with a $N = 2$ extended supersymmetry applied to electroweak and strong interactions, or with a grand-unification symmetry like $SU(5)$, $O(10)$ or $E(6)$, before moving to higher dimensions [45, 65].

B. Are the $N = 2$ “central charges” really central?

Within $N = 2$ supersymmetry the particles get again organized within massless or massive multiplets, the W^\pm and Z , $X^{\pm 4/3}$ and $Y^{\pm 1/3}$ gauge bosons belonging to different kinds of massive gauge multiplets.

To start with, $N = 2$ theories involve a new sort of massive multiplet known as an “hypermultiplet”, describing massive spin- $\frac{1}{2}$ and spin-0 particles, with the two spin-0 fields transforming as the two components of a $SU(2)_R$ isodoublet while the Dirac spinor is an isosinglet [43]. This massive multiplet with maximum spin- $\frac{1}{2}$ looks at first intriguing as in principle it should not exist, not being a representation of the $N = 2$ supersymmetry algebra $\{Q^i, \bar{Q}^j\} = -2P\delta^{ij}$.

The theory being nevertheless invariant under the two supersymmetry generators Q_1 and Q_2 , and thus under $\{Q^1, \bar{Q}^2\}$, the above $N = 2$ algebra must be modified, to allow for additional bosonic symmetry generators within the expression of $\{Q^1, \bar{Q}^2\}$. How it gets modified is quite interesting, especially in view of the spontaneous breakings of the grand-unification symmetry, in a way allowing for the electroweak breaking to occur.

We are led to consider, in a way compatible with Lorentz symmetry (and the symmetry property of the anticommutators) the extended algebra

$$\{Q^i, \bar{Q}^j\} = -2P \delta^{ij} + 2\epsilon^{ij}(Z + \gamma_5 Z'), \quad (139)$$

where one should still identify correctly the two symmetry generators Z and Z' . They are often referred to as “central charges”, meaning that they ought to commute with all other symmetry generators of the theory.

Let us consider, however, a R -symmetric theory under which Q^1 and Q^2 transform chirally according to (35),

$$Q^i \xrightarrow{R} e^{-\gamma_5 \alpha} Q^i, \quad (140)$$

or equivalently $Q_R^i \rightarrow e^{i\alpha} Q_R^i$. The operators Z and Z' appearing in (139) get rotated according to

$$Z - iZ' \xrightarrow{R} e^{2i\alpha} (Z - iZ'), \quad (141)$$

so that [45]

$$[R, Z] = -2iZ', \quad [R, Z'] = 2iZ. \quad (142)$$

These operators Z and Z' , although commonly referred to as “central charges”, do not belong to the center of the symmetry algebra!

This may seem surprising in view of the study of all possible supersymmetries of the S matrix, according to which Z and Z' must commute with all symmetry generators [73]. This analysis, however, disregards massless particles and symmetry breaking. It is thus not directly applicable here, both massless particles and symmetry-breaking effects playing an essential role. The Z and Z' generated from the anticommutation relations (139) do not necessarily commute with all symmetry generators, as seen in (141,142) for R symmetry [45]. Z and Z' do not in general commute between themselves nor with gauge symmetry generators, in the non-abelian case, and as such do not qualify as “central charges”.

Indeed in a gauge theory the anticommutators of the two supersymmetry generators may be expressed as in (139) but only up to (non-abelian or abelian) field-dependent gauge transformations (and modulo field equations of motion). For a $N = 2$ supersymmetric Yang-Mills theory equation (139) reads [44]

$$\{Q_R^1, \overline{Q_L^2}\} = 2 \frac{1+i\gamma_5}{2} \underbrace{g T_i (a_i - ib_i)}_{Z - iZ'}, \quad (143)$$

a_i and b_i being the spin-0 partners of the adjoint or singlet gauge fields V_i^μ , so that

$$Z = g T_i a_i, \quad Z' = g T_i b_i. \quad (144)$$

The T_i denote the generators of the gauge symmetry group. a_i and b_i , described by adjoint or singlet chiral superfields with $R = 2$ (as for the nMSSM singlet S in (52)), transform under R according to

$$a_i - ib_i \xrightarrow{R} e^{2i\alpha} (a_i - ib_i), \quad (145)$$

so that Z and Z' do actually transform according to (141).

The spin-0 adjoints a_i and b_i will soon be interpreted as originating from the 5th and 6th components $V^5 = a$ and $V^6 = b$ of the 6d gauge fields $V^{\hat{\mu}}$. The R transformation (145) gets then associated with a 6d rotation \mathcal{R}_{56} in compact space, under which

$$V_i^5 - iV_i^6 \xrightarrow{\mathcal{R}_{56}} e^{2i\alpha} (V_i^5 - iV_i^6), \quad (146)$$

$$P^5 - iP^6 \xrightarrow{\mathcal{R}_{56}} e^{2i\alpha} (P^5 - iP^6), \quad (147)$$

and similarly for the extra components of the covariant translation operator $\mathcal{P}^{\hat{\mu}}$ in 6 dimensions.

The translation of these adjoint gauge scalars (constrained to $f_{ijk} < a_j > < b_k > = 0$ for the potential to be minimum) leads to a spontaneous breaking of the non-abelian gauge symmetry. It generates in the anticommutation relations (139) finite field-independent parts

$$<Z> = g T_i <a_i> \quad \text{and} \quad <Z'> = g T_i <b_i>. \quad (148)$$

These now truly deserve the name of central charges, commuting between themselves and with all unbroken symmetry generators [44, 45]

$$<Z> \text{ and } <Z'> \text{ are the central charges.} \quad (149)$$

This leads to the *spontaneous generation of central charges* in the anticommutation relations of the $N = 2$ supersymmetry algebra, i.e. to a *spontaneous modification of the graded symmetry algebra*. Note that a central charge Z_0 may already be present before spin-0 fields get translated. These symmetry operators act as abelian, commuting in particular with all gauge symmetry generators surviving the spontaneous breaking. In practice we shall often drop the symbols $<>$ and simply refer for convenience to Z and Z' , instead of $<Z>$ and $<Z'>$, as being the central charges.

C. Solving the “doublet-triplet splitting problem”

Central charges can thus be “spontaneously generated” in a $N = 2$ supersymmetry algebra, one of them at least being closely connected with the spontaneous breaking of the grand-unification symmetry [44, 45]. It could be the weak-hypercharge operator Y , spontaneously generated in the algebra through the symmetry breaking $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$, in a way which preserves the rank of the gauge group.

But Y is not conserved by the electroweak breaking. And, conversely, this breaking cannot occur in a $N = 2$ theory with a central charge proportional to Y . This is easily seen as initially massless spin-0 BEH doublets with $Y = \pm 1$ would acquire large masses $\frac{3}{5}m_X$, getting unable to trigger the electroweak breaking.

But we can start with initially massive quintuplets of mass m before the grand-unification breaking, and take advantage of the *flat directions for the adjoint or singlet gauge scalars* a_i and b_i in a $N = 2$ theory. Indeed the adjoint mass parameter vanishes in the superpotential, already as a consequence of R symmetry, as for the singlet S in the nMSSM. The magnitude of the adjoint v.e.v., denoted by V , can then freely adjust in the weak-hypercharge direction preserving $SU(3) \times SU(2) \times U(1)$, so that the resulting doublet mass parameter $m_D = m - \frac{3}{2}gV$ vanishes. This vanishing allows for the electroweak breaking, with the triplet mass parameter $m + gV$ getting identical to $m_X = \frac{5}{2}gV$ [45]:

$$\boxed{\begin{array}{l} GUT\ br. \\ \xrightarrow{m_{\text{quint.}}} \end{array} \left\{ \begin{array}{l} m_D = m - \frac{3}{2}gV \equiv 0, \\ m_T = m + gV = \frac{5}{2}gV \equiv m_X. \end{array} \right.} \quad (150)$$

This mechanism provides an automatic and natural solution to the “doublet-triplet splitting problem”. This one is usually considered as a serious difficulty for the electroweak breaking in a $N = 1$ supersymmetric grand-unified theory (and even more in a non-supersymmetric one), requiring a very large and unnatural adjustment for parameters of the order of the grand-unification scale. This severe fine-tuning problem is solved by moving to $N = 2$, and from there to higher-dimensional theories. The vanishing of the doublet mass parameter m_D allows for their translation generating the electroweak breaking. Even better, this translation requires, conversely, that the doublet mass parameter m_D vanishes exactly *by locking it to 0, for the energy to be minimum*.

D. The massive $X^{\pm 4/3}$, $Y^{\pm 1/3}$, W^\pm and Z multiplets, within $N = 2$ supersymmetry

All spin-1 gauge bosons must then belong to massless or massive multiplets of $N = 2$ supersymmetry. But there are *three different types of massive gauge multiplets*, in contrast with $N = 1$. Type I multiplets are appropriate to describe the W^\pm and Z as in (129,133). They involve no central charge, and may be complex or real. Type II and type III multiplets, on the other hand, have a non-vanishing value of the central charge $\langle Z \rangle$ (from now on simply denoted by Z) and are necessarily complex. They differ by their field content, and are appropriate to the description of grand-unification gauge bosons such as $X^{\pm 4/3}$ and $Y^{\pm 1/3}$, in a $SU(5)$ theory.

These grand-unification bosons, which have the same weak-hypercharge $Y = \pm 5/3$, belong to two multiplets with the same value of the central charge

$$Z(X^{\pm 4/3}) = Z(Y^{\pm 1/3}) = \pm m_X \quad (151)$$

(up to a possible convention-dependent sign). This one

includes a contribution $\pm \frac{3}{5} m_X Y$, spontaneously generated [44] in the $N = 2$ algebra when the grand-unification symmetry is spontaneously broken to $SU(3) \times SU(2) \times U(1)$. This central charge reads

$$Z = Z_0 + \frac{3}{5} m_X Y, \quad (152)$$

and is such that

$$Z(\text{spin-0 doublets}) \equiv 0, \quad (153)$$

as in (150), naturally allowing for the doublet translations responsible for the electroweak breaking.

The $X^{\pm 4/3}$ belongs to a smaller multiplet, of type II, including a single spin-0 boson $x^{\pm 4/3}$ and two Dirac xino (anti)triplets, with $m_X = |Z| > 0$. The $Y^{\pm 1/3}$ belongs to a larger multiplet, of type III, with 5 spin-0 bosons and 4 Dirac yino (anti)triplets, verifying altogether [74]

$$\left\{ \begin{array}{ll} \text{type II :} & m_X = |Z| > 0, \\ \text{type III :} & m_Y > |Z| = m_X > 0. \end{array} \right. \quad (154)$$

The smaller character of the $X^{\pm 4/3}$ multiplet as compared to the $Y^{\pm 1/3}$ one is associated with the mass equality $m_X = |Z|$, in contrast $m_Y > |Z|$. This may be easily understood when moving to 6 dimensions, where the $X^{\pm 4/3}$ is massless in relation with an unbroken $SU(4)$ *electrostrong symmetry* in 6 dimensions while the $Y^{\pm 1/3}$ is already massive (with mass m_W) in 6 dimensions.

The $Y^{\pm 1/3}$ multiplet, larger than the $X^{\pm 4/3}$ one, also accommodates the 4 triplet components from the 4 quintuplets. All of them have the same mass² m_Y^2 , including a m_X^2 contribution in agreement with (150). These 4 quintuplets describe in particular the 4 spin-0 doublets responsible for the electroweak breaking in a $N = 2$ theory, also providing spin-0 partners for the W^\pm and Z bosons.

The $Y^{\pm 1/3}$ (and associated partner) mass² originates from the two contributions generated by the adjoint and doublet v.e.v.’s V and v , respectively, so that

$$m_Y^2 = \underbrace{m_X^2}_{Z^2} + m_W^2. \quad (155)$$

The W^\pm and Z , on the other hand, carry no central charge Z and belong to massive gauge multiplets of type I, describing 4 inos and 5 spin-0 bosons for every spin-1 particle, as in (129,133).

When supersymmetry is broken mass relations similar to (138), like

$$\left\{ \begin{array}{l} m^2(\text{xinos}) = m_X^2 + m_{3/2}^2, \\ m^2(\text{yinos}) = m_Y^2 + m_{3/2}^2 = m_X^2 + m_W^2 + m_{3/2}^2, \end{array} \right. \quad (156)$$

are obtained for xinos and yinos, and interpreted in terms of momenta $\pm m_{3/2}$ carried along an extra compact dimension [46].

IX. SUPERSYMMETRY AND GRAND-UNIFICATION IN EXTRA DIMENSIONS

A. From $N = 2$ supersymmetry to 6 dimensions

These $N = 2$ theories may then be formulated in a 5 or 6 dimensional space-time [46, 65], with the “central charges” Z and Z' getting turned into the 5th and 6th components of the (covariant) momentum along the compact dimensions [44]. The two spin-0 photons and spin-0 “sgluon” octets present in $N = 2$ supersymmetry get described by the fifth and sixth components of the photon and gluon fields V_i^μ in 6 dimensions. *The W^\pm and Z masses are already present in 6 dimensions*, where the photon and gluons are coupled with the same strength.

Viewing $Q\sqrt{3/8}$ as one of the $SU(4)$ *electrostrong symmetry* generators, suitably normalized in the same way as for the $SU(3)$ generators, provides in 6d the $SU(4)$ relation between the electromagnetic and strong couplings,

$$\begin{aligned} \text{electrostrong symmetry} &\implies \\ e_{6d} &= \sqrt{\frac{3}{8}} g_{36d}, \quad \text{i.e. } \alpha_{6d} = \frac{3}{8} \alpha_{36d}. \end{aligned} \quad (157)$$

This relation is exact in 6d as long as we do not introduce the grand-unification breaking through antiperiodic boundary conditions for GUT-odd particles, discussed in the next subsection. We also have, by returning to $SU(5)$ to include weak in addition to electrostrong interactions, $\sin^2 \theta = e^2/g^2 = 3/8$ for the electroweak angle in 6d, at the classical level.

The central charge Z of the $N = 2$ algebra in 4 dimensions, essential to the discussion of the grand-unification breaking, originates from the fifth component of the (covariant) momentum along a compact dimension, according to

$$\mathcal{P}^5 = - \left(Z_\circ + \frac{3}{5} m_X Y \right) \quad (158)$$

(up to a possible convention-dependent sign). Once we are in 5 or 6 dimensions, we only have to refer to the extra components of the covariant momenta, \mathcal{P}^5 and \mathcal{P}^6 , rather than to the corresponding central charges Z and Z' of the 4d $N = 2$ theory.

The particle content of the $N = 2$ multiplets are given in [45, 46]. The massive gauge multiplet (129) describing the Z in 4d originates from the massive Z gauge multiplet in 6d, such that

Z	$\overset{6d}{\underset{SUSY}{\iff}}$	8-comp. Dirac zino	$\overset{6d}{\underset{SUSY}{\iff}}$	3 spin-0 bosons.
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(159)

This multiplet reduces to (129) in 4 dimensions, in which the Z is associated with 4 Majorana zinos and 5 spin-0 BEH bosons. Similar expressions hold for the W^\pm and $Y^{\pm 1/3}$ multiplets. This set of 5 neutral spin-0 BEH bosons associated with the Z in 4 dimensions, before the breaking of the $N = 2$ supersymmetry, is similar to the nMSSM one in (129) for $\lambda = \sqrt{(g^2 + g'^2)/2} \simeq .52$, and presumably includes the 125 GeV boson found at CERN.

$N = 2$ theories in 4 dimensions, being vectorlike, also include mirror partners for quarks and leptons, to which they are coupled through the exchanges of spin-0 gluons and photons, in particular. But no such particles have been observed yet. Their presence at low energy may be avoided by considering a mirror-parity operator M_p under which mirror particles, as well as spin-0 quarks and photons, ..., are M_p -odd. The Z multiplet then gets further reduced, to include just a single spin-0 boson z associated with the Z .

Indeed among the 4 spin-0 doublets (originating from 4 quintuplets) the usual ones h_1 and h_2 are taken as M_p -even so as to survive in the low-energy theory. Their $N = 2$ partners h'_1 and h'_2 , being M_p -odd, disappear from the low-energy theory. The definition of this M_p operator involves in particular, as seen from (145-147), a rotation $\mathcal{R}_{56}(\pi)$, equivalent to a reflexion symmetry in compact space, $x^5 \rightarrow -x^5$, $x^6 \rightarrow -x^6$, under which

$$V_i^\mu \rightarrow V_i^\mu; \quad V_i^5 \rightarrow -V_i^5, \quad V_i^6 \rightarrow -V_i^6. \quad (160)$$

Anticipating on the supersymmetry breaking discussed in the next subsection, the R -odd zinos are present only at the compactification scale associated with the sixth dimension, starting with two Dirac zinos (combining a Dirac gaugino with a Dirac higgsino), at mass² [46]

$$m^2(\text{zinos}) = m_Z^2 + \pi^2/L_6^2. \quad (161)$$

We then simply remain, in the low-energy 4d theory below the compactification scales, with the field-content of the standard model but for the presence of the two spin-0 doublets h_1 and h_2 , with quartic doublet couplings fixed by $(g^2 + g'^2)/8$ and $g^2/2$ as in (5,6,63). This is crucial for the gauge-BEH unification according to which [1, 7, 8]

$$\text{spin-1 } Z \quad \overset{\text{SUSY}}{\longleftrightarrow} \quad \overset{\text{SUSY}}{\longleftrightarrow} \quad \text{spin-0 BEH boson } z,$$

(162)

with the spin-0 z having the same mass as the Z before supersymmetry breaking effects get taken into account. We also expect, in the same way, the following association

$$\text{spin-1 } W^\pm \quad \overset{\text{SUSY}}{\longleftrightarrow} \quad \overset{\text{SUSY}}{\longleftrightarrow} \quad \text{spin-0 BEH boson } w^\pm,$$

(163)

for the charged spin-0 boson in (85),

$$w^\pm \equiv H^\pm = \sin \beta h_1^\pm + \cos \beta h_2^\pm. \quad (164)$$

In a grand-unified theory, with for example $SU(5)$ as the gauge group, this one gets spontaneously broken down to an $SU(4)$ electrostrong symmetry group. The $X^{\pm 4/3}$ (anti)triplet remains massless in 6d, where the $Y^{\pm 1/3}$ (anti)triplet has the same mass m_W as the W^\pm , with which they form a

$$SU(4) \text{ electrostrong antiquartet} \quad \begin{pmatrix} Y^{+1/3} \\ W^- \end{pmatrix}. \quad (165)$$

In this higher-dimensional space-time, the $SU(5)$ symmetry is broken through the BEH-quintuplet v.e.v.'s, providing in 6 dimensions equal masses to the $Y^{\pm 1/3}$ and W^\pm gauge fields, according to

$SU(5)$	$\xrightarrow{\text{EW breaking in 6d}}$	$SU(4) \text{ electrostrong gauge group},$
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(166)

leading to the $SU(4)$ relation $\alpha_{6d} = (3/8) \alpha_{s6d}$ in 6 dimensions, and to $\sin^2 \theta = 3/8$, at the classical level.

This electrostrong-weak breaking in 6d *separating weak from electrostrong interactions* leads in 4d to the mass relation (155)

$$m_Y^2 = m_X^2 + m_W^2 \quad (167)$$

found previously, with $m_X^2 = P_5^2 + P_6^2$, for the X and Y gauge bosons and their susy partners. These relations are valid for each excitation level of the extra compact dimensions, for the $X^{\pm 4/3}$ and $Y^{\pm 1/3}$ gauge fields in 6 dimensions.

B. Grand-unification and supersymmetry breaking from discrete boundary conditions in compact space

The extra compact space dimensions may play an essential role in the breaking of the supersymmetry and grand-unification symmetries, together with the mirror-parity operator M_p allowing to avoid mirror quarks and leptons, spin-0 gluons and photons, and additional spin-0 BEH bosons, in the low-energy spectrum. This may be done through boundary conditions involving, in an interesting way, *discrete* rather than continuous symmetries. They include R -parity, a *GUT*-parity G_p and mirror parity M_p acting as translation and reflexion symmetries in the compact space, thanks to its topological properties [46, 65].

These three discrete symmetries naturally allow for the presence at low energy of the two spin-0 doublets h_1 and h_2 , even under R_p , G_p and M_p . They can thus generate the same spontaneous electroweak breaking in 4 dimensions as already resulting in (159,166) from the grand-unification breaking into the $SU(4)_{es}$ electrostrong symmetry subgroup in 6 dimensions, with the W^\pm and Z

masses in 4 dimensions directly originating from the 6d theory.

The breaking of the supersymmetry may be obtained by identifying the action of travelling along a complete loop \mathcal{L}_6 in compact space (i.e. a translation $x^6 \rightarrow x^6 + L_6$, in the simplest example of a flat torus) with a discrete R -parity transformation, $R_p = (-1)^{3(B-L)} (-1)^{2S} = \pm 1$:

$\text{travelling along a complete loop } \mathcal{L}_6 \text{ in compact space}$ $\equiv R\text{-parity transformation}.$

(168)

This makes all superpartners naturally “very heavy”, i.e. at the compactification scale:

$$m(R\text{-odd superpartners}) \approx \text{compactification scale}. \quad (169)$$

This compactification scale is unknown but has to be \gtrsim TeV scale, at least. We may thus be lucky enough to see superpartners, together with the opening of extra space dimensions, in a not-too-distant future. But we may also have to face the eventuality that superpartner masses be considerably larger than the presently accessible \approx TeV scale, especially if the compactification of extra dimensions also sets the scale for the grand-unification breaking. The latter scale, however, may be substantially reduced as compared to usual expectations, as we shall see.

In a similar way, the breaking of the $SU(4)$ electrostrong symmetry group in 6d may be obtained by identifying travelling along a complete loop \mathcal{L}_5 (e.g. a translation $x^5 \rightarrow x^5 + L_5$ on a flat torus) with a discrete Z_2 *GUT*-parity transformation G_p ,

$\text{travelling along another loop } \mathcal{L}_5 \text{ in compact space}$ $\equiv GUT\text{-parity transformation},$
--

(170)

This one is defined from expression (152) of the central charge Z , as $e^{i\pi Z/m_X}$, or more precisely as

$$\text{GUT-parity } G_p = G' \times e^{i\pi \frac{3}{5}Y} = (-1)^{Z/m_X} = \pm 1. \quad (171)$$

Here $G' = e^{i\pi Z_0/m_X}$ is a global symmetry operator commuting with both $SU(5)$ and supersymmetry, acting in particular on quark and lepton grand-unification multiplets, and spin-0 BEH multiplets. G_p may be expressed in terms of the central charge Z present in the $N = 2$ supersymmetry algebra in 4 dimensions, as in (139,151-153), part of which, proportional to the weak-hypercharge Y , is generated spontaneously during the breaking of the grand-unification symmetry [44, 45]. Taking the fifth dimension as cyclic, of size L_5 , with periodic and antiperiodic boundary conditions for *GUT*-even and *GUT*-odd fields, we can identify the action of a *GUT*-parity transformation with the one of a translation of L_5

on the torus, so that

$$\begin{aligned} & \text{action of GUT-parity} \\ & \equiv \text{action of } e^{i P_5 L_5} = (-1)^{P_5 / \frac{\pi}{L_5}}. \end{aligned} \quad (172)$$

The $X^{\pm 4/3}$ and $Y^{\pm 1/3}$ gauge bosons, with $Y = \pm 5/3$, are odd under GUT-parity, and carry momenta $(2n_5 + 1)\pi/L_5$ along the fifth dimension. This is also the case for the spin-0 triplet partners of the electroweak doublets within $SU(5)$ quintuplets. These belong to the same massive gauge multiplet of type III as the $Y^{-1/3}$, in agreement with (151,171) [45, 46, 74].

This *triplet-doublet splitting mechanism*, already operating within $N = 2$ supersymmetric GUTs as in (150) [45], is of the same nature as the one splitting the $X^{\pm 4/3}$ and $Y^{\pm 1/3}$ masses away from the gluon, photon, W^\mp and Z masses. It provides for their physical spin-0 triplet and doublet components the same masses as for the $Y^{\pm 1/3}$, W^\mp and Z , before the supersymmetry breaking. The massless components associated with the would-be Goldstone bosons are eliminated when the $Y^{\pm 1/3}$, W^\mp and Z acquire their 6d masses m_W , m_W and m_Z .

The $X^{\pm 4/3}$, $Y^{\pm 1/3}$ and color-(anti)triplet spin-0 bosons, being G_p -odd, have no direct couplings between two ordinary (anti)quarks or (anti)leptons, even under GUT-parity. This is in contrast with ordinary GUTs, and implies that

$$\text{the proton is expected to be stable}, \quad (173)$$

at least in the simplest situations [65, 75]. The corresponding compactification scale associated with the grand-unification breaking might then be lower, and possibly significantly lower, than the $\approx 10^{16}$ GeV usually considered.

Altogether the spontaneous breaking of the supersymmetry and grand-unification symmetries may both be induced through the compactification of the extra dimensions. This leads to the possibility of fixing the scales associated with these breakings in terms of the compactification scales for the extra dimensions [46]. In the simplest case of two flat extra dimensions and for the lowest-lying excited states, we would get relations like

$$\left\{ \begin{array}{l} m_{3/2} = \frac{\pi}{L_6} = \frac{1}{2R_6} \\ \quad (\text{from } R\text{-parity} \equiv \text{translation of } L_6), \\ m_X = \frac{\pi}{L_5} = \frac{1}{2R_5} \\ \quad (\text{from GUT-parity} \equiv \text{translation of } L_5), \end{array} \right. \quad (174)$$

up to radiative corrections. The lowest-lying superpartners, or grand-unification particles, are expected to be present at these mass scales determined by $m_{3/2}$ and m_X , respectively.

This use of *discrete* boundary conditions associated with a non-trivial topology, involving for supersymmetry R -parity rather than a continuous symmetry, allows to link rigidly these fundamental supersymmetry and grand-unification breaking parameters $m_{3/2}$ and m_X to the compactification scales. This approach contrasts with the initial one in [76] disregarding fields corresponding to excited states that become infinitely massive when the size of the compact space is shrunk to zero (in particular states of masses proportional to π/L , essential here). We obtain instead *quantized mass parameters* fixed in terms of the compactification scales, with the *geometry* now determining the masses of the new particles in which we are interested.

C. Implications for the compactification scales

The resulting 4d theory has, in its simplest version, *the same content as the standard model* at low-energy *but for the two spin-0 doublets h_1 and h_2* , while still allowing for the gauge/BE-Higgs unification that is one of the most interesting features of supersymmetric theories. The new (sixth) dimension opens up at the compactification scale $m_{3/2}$, i.e. π/L_6 in the simplest case. Superpartners, mirror particles, spin-0 gluons etc., as for a $N = 2$ theory in 4 dimensions [45], would then appear at or above this threshold. They now originate from a $N = 1$ theory in 5 dimensions, its mass spectrum resulting from the discrete boundary conditions involving the R_p and M_p symmetries.

Let us assume $m_{3/2}$ smaller than m_X . Below $m_{3/2}$ the theory has in its simplest version the same field content as the standard model, but for the second spin-0 doublet. The evolution of the gauge couplings (or simply of the differences $g_i^{-2} - g_j^{-2}$) in the 4d theory between m_W and $m_{3/2}$ is only slightly modified as compared to the standard model. It does not lead a grand-unification of these three couplings below the compactification scale.

Above $m_{3/2}$ the theory gets 5-dimensional, non-renormalisable, with gauge couplings having the dimension of $\text{mass}^{-1/2}$. We can no longer discuss as usual the running of the gauge couplings. One may still feel tempted to continue evaluating an evolution of effective couplings with energy, taking into account only a finite number of states up to a cut-off mass Λ , but one should be cautious before drawing conclusions. In addition the asymptotic freedom of QCD is expected to be ruined owing to the extra mirror families of quarks and leptons, unless one considers that quarks, leptons and mirror partners do not have excited states for the compact dimensions.

We recall that the dimension (x^6) responsible for the evolution of (effective) gauge couplings between $m_{3/2}$ and m_X is distinct from the one (x^5) responsible for the breaking of the GUT symmetry at the higher grand-unification scale. The latter, $m_X = \pi/L_5$, however, may be only slightly larger than $m_{3/2} = \pi/L_6$.

$\sin^2 \theta$, evaluated in a 4d theory, is in particular sensitive to the number of spin-0 doublets and associated higgsinos, usually h_1, h_2 and \tilde{h}_1, \tilde{h}_2 in a $N = 1$ theory (counting very much as for 6 spin-0 doublets) with the field content of the (N/n)MSSM. Only the 2 doublets h_1 and h_2 , without their accompanying higgsinos, are here present in the 4d theory below $m_{3/2}$. The evolution of the three gauge couplings, if extrapolated up to a unified value within the 4d theory, would necessitate a too small $\sin^2 \theta$, much as for the standard model.

This may be more than compensated, however, by the 4 spin-0 doublets h_1, h_2, h'_1, h'_2 + associated higgsinos (twice as much as in the MSSM) present in the 5d theory between $m_{3/2}$ and m_X . These extra doublet degrees of freedom (taking also into account the extra adjoint gaugino and spin-0 fields) tend to lead to a too large value of $\sin^2 \theta$. This indicates that the correct value of $\sin^2 \theta$ may be obtained from a balance between these two effects, presumably with $m_{3/2}$ not far below m_X .

If the proton is indeed stable m_X may be much lower than the usual $\approx 10^{16}$ GeV scale, especially with a faster evolution of effective gauge couplings in the 5d theory between $m_{3/2}$ and m_X . Their unification may then occur for a rather low value of the grand-unification scale m_X . One may even imagine that these unification and compactification scales

$$m_{3/2} = \frac{\pi}{L_6} < m_X = \frac{\pi}{L_5} \quad (175)$$

may be not so far above the \approx few TeV scale soon accessible at LHC. This would provide new perspectives for a possible discovery of superpartners, new space dimensions and maybe grand-unification particles, in a not-too-distant future.

This set of jointly-operating mechanisms, based on supersymmetry, extra dimensions and discrete symmetries, allows for the electroweak breaking to already occur in 6d dimensions, where it leaves unbroken an electrostrong symmetry group. It provides in 4d the electroweak breaking induced by h_1 and h_2 at low energies, even in the presence of significantly larger scales associated with grand-unification and possibly (in a more remarkable way) supersymmetry breaking. In particular

*no fine-tuning between GUT-scale parameters
is required,*

(176)

and the electroweak breaking in the low-energy theory appears largely insensitive to the behavior of the higher-dimensional theory.

To each of the three conserved symmetries R_p , G_p and M_p acting in compact space is associated a stable particle, possible candidate for the non-baryonic dark matter of the Universe. The LSP and LGP, lightest supersymmetric and lightest grand-unification particles, are

directly associated with the excitation of the compact sixth and fifth dimensions. The mirror-parity operator M_p , associated with the reflexion of the compact coordinates (or rotation of π in compact space) leads to the lightest M -odd particle or LMP. This one, to be found among mirror quarks and leptons, and spin-0 gluons and photons or other neutral spin-0 gauge bosons, ..., is also associated with the excitation of the compact dimensions.

X. CONCLUSION

In addition to superpartners, supersymmetric theories lead to an extended set of spin-0 bosons H^\pm, H, h, A, \dots . Some of them appear as extra states for massive spin-1 gauge bosons, providing a relation between spin-1 mediators of gauge interactions and spin-0 particles associated with symmetry breaking and mass generation.

Searches for supersymmetric particles started in the late seventies, first looking for light gluinos and associated R -hadrons, light charged sleptons, etc., often relying on the missing-energy momentum carried away by unobserved neutralino or gravitino LSP's, at the modest energies accessible at the time [3, 77]. Considerable work has been done since throughout the world, most notably at PETRA (DESY) and PEP (SLAC), LEP (CERN) and at the Tevatron (Fermilab). These searches are now at the forefront of particle physics with the restart of LHC experiments at CERN.

All this could not be discussed here, nor the status of the lightest supersymmetric particle, presumably a neutralino, as a possible dark matter candidate in a R -parity conserving theory. We know now that strongly-interacting squarks and gluinos should be heavier than about 1 TeV at least. We refer the reader to the original results from the ATLAS and CMS collaborations at LHC [34, 35, 68, 69], and to the other articles in this book to complete this theoretical description with the presentation of experimental results and constraints on supersymmetric particles and additional spin-0 BEH bosons.

The next run of LHC experiments, with an energy increased from 8 to 13 TeV, may well allow for the direct production of supersymmetric particles, and of an extended system of spin-0 bosons including a charged H^\pm . Will this energy be sufficient, and at which energy scale should the new superpartners be found? Is it indeed not too far from the electroweak scale, and accessible at LHC? Or still significantly larger, as it could happen for superpartner masses determined by the very small size of an extra dimension ($L \lesssim 10^{-17}$ cm corresponding to $\pi\hbar/Lc \gtrsim 6$ TeV/ c^2)?

In any case the 125 GeV boson observed at CERN may well be interpreted, up to a mixing angle induced by supersymmetry breaking, as the spin-0 partner of the

Z under two supersymmetry transformations,

$$\text{spin-1 } Z \xleftrightarrow{\text{SUSY}} \xleftrightarrow{\text{SUSY}} \text{spin-0 BEH boson}, \quad (177)$$

i.e. as a Z that would be deprived of its spin. This provides within a theory of electroweak and strong interactions the first example of two known fundamental particles of different spins that may be related by super-

symmetry.

Even if R -odd superpartners were still to remain out of reach for some time, possibly due to large momenta along very small space dimensions, supersymmetry could still be tested in the gauge-and-BEH sector at present and future colliders, in particular through the properties of the new spin-0 boson and the search for additional ones.

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